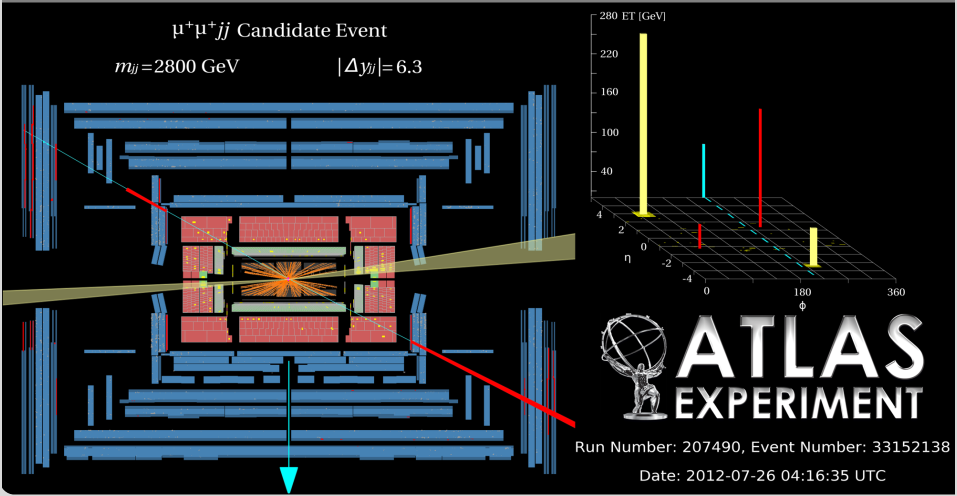
Karlsruhe Institute of Technology

NLO QCD corrections to *VVjj* production at the LHC

DESY-HU seminar
Francisco Campanario, Matthias Kerner, LE Duc Ninh, Dieter Zeppenfeld | June 19 2014



Outline

- *VVjj* production (with leptonic decays) at the LHC: motivation
- *VVjj* @ NLO QCD : some calculational details
- Phenomenological results: WZ, $W\gamma$, ZZ, W^+W^+
- Summary

VVjj production at the LHC: why?

- Motivation:
  - Sensitive to $VV \to VV$ scattering, quartic quage-boson couplings.
  - Important background for new physics searches.

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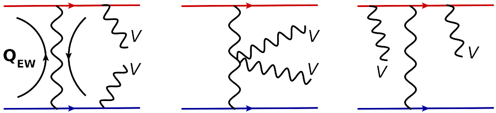
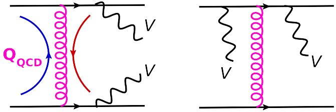
- Motivation:
  - Sensitive to $VV \to VV$ scattering, quartic quage-boson couplings.
  - Important background for new physics searches.
- Classification at LO: 2 mechanisms
  - EW mechanism (vector boson fusion, VBF): $\sigma_{EW} \propto \alpha^6$

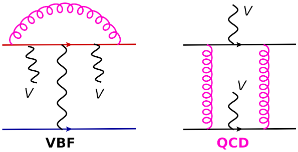


  - QCD mechanism: $\sigma_{QCD} \propto \alpha_s^2 \alpha^4$



  - Interference: color and kinematically suppressed.
    $\rightsquigarrow$ can be neglected for a-few-percent precision measurements at the LHC.

What have been done at NLO QCD?



- EW mechanism (VBF): consider QCD corrections to each quark lines separately $\rightsquigarrow$ pentagons at most.
  - W^+W^-jj: [Jager, Oleari, Zeppenfeld, 2006]
  - $ZZjj$: [Jager, Oleari, Zeppenfeld, 2006]
  - $W^{\pm}Zjj$: [Bozzi, Jager, Oleari, Zeppenfeld, 2007]
  - $W^{\pm}W^{\pm}jj$: [Jager, Oleari, Zeppenfeld, 2009], [Denner, Hosekova, Kallweit, 2012]
  - $W^{\pm}\gamma jj$: [Campanario, Kaiser, Zeppenfeld, 2013]

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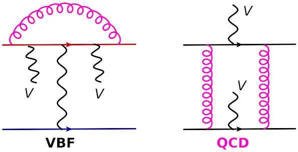

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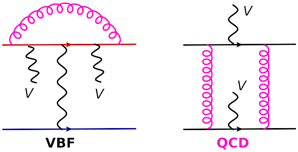

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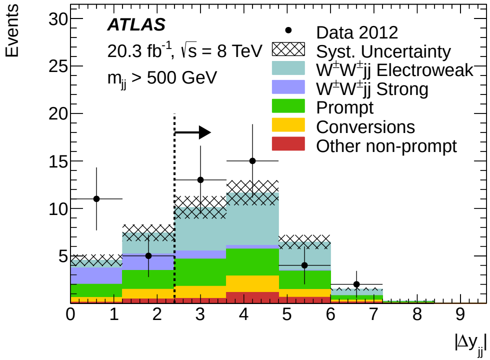
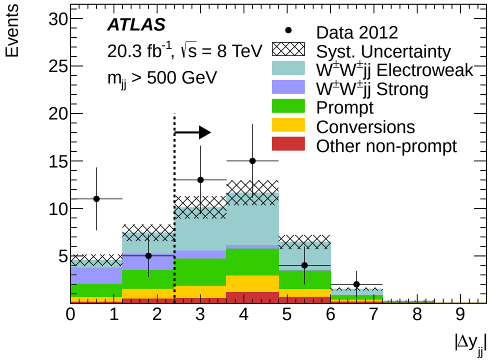
- almost all processes (notable exeption: QCD W^+W^-jj) are included in VBFNLO program.
- Same-sign $W^\pm W^\pm jj$ are special: clean signal, small background (no $t\bar{t}$), simplest calculation.

Same-sign $W^{\pm}W^{\pm}jj$@ATLAS, 2014



	Inclusive Region			VBS Region		
	$e^{\pm}e^{\pm}$	$e^{\pm}\mu^{\pm}$	$\mu^{\pm}\mu^{\pm}$	$e^{\pm}e^{\pm}$	$e^{\pm}\mu^{\pm}$	$\mu^{\pm}\mu^{\pm}$
Prompt	3.0 ± 0.7	6.1 ± 1.3	2.6 ± 0.6	2.2 ± 0.5	4.2 ± 1.0	1.9 ± 0.5
Conversions	3.2 ± 0.7	2.4 ± 0.8	–	2.1 ± 0.5	1.9 ± 0.7	–
Other non-prompt	0.61 ± 0.30	1.9 ± 0.8	0.41 ± 0.22	0.50 ± 0.26	1.5 ± 0.6	0.34 ± 0.19
$W^{\pm}W^{\pm}jj$ Strong	0.89 ± 0.15	2.5 ± 0.4	1.42 ± 0.23	0.25 ± 0.06	0.71 ± 0.14	0.38 ± 0.08
$W^{\pm}W^{\pm}jj$ Electroweak	3.07 ± 0.30	9.0 ± 0.8	4.9 ± 0.5	2.55 ± 0.25	7.3 ± 0.6	4.0 ± 0.4
Total background	6.8 ± 1.2	10.3 ± 2.0	3.0 ± 0.6	5.0 ± 0.9	8.3 ± 1.6	2.6 ± 0.5
Total predicted	10.7 ± 1.4	21.7 ± 2.6	9.3 ± 1.0	7.6 ± 1.0	15.6 ± 2.0	6.6 ± 0.8
Data	12	26	12	6	18	10

Same-sign $W^\pm W^\pm jj$@ATLAS, 2014



- Total number of events: 50 (INC) and 34 (VBS)
- Data vs. background (significance): 4.5σ (INC, signal = QCD + EW), 3.6σ (VBS, signal = EW)

	Inclusive Region			VBS Region		
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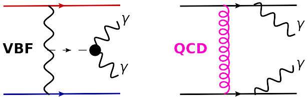
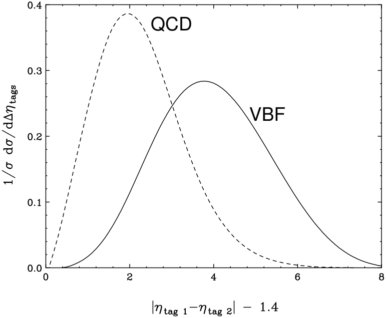
VBF vs. QCD background: a Higgs example

[Rainwater, hep-ph/9908378]



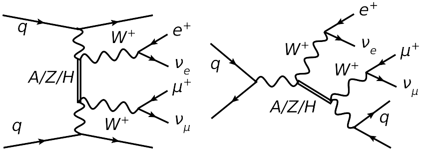
LHC $\sqrt{s} = 14$ TeV, $p_{Tj} \geq 20$ GeV, $|\eta_j| < 5$, $\Delta R_{jj} \geq 0.7$, $|\eta_\gamma| < 2.5$, $\Delta R_{j\gamma} \geq 0.7$;
$\eta_{j,min} + 0.7 < \eta_\gamma < \eta_{j,max} - 0.7$, $\eta_{j_1}\eta_{j_2} < 0$



The two tagging jets are more separated in VBF than in QCD background!

$W^{\pm}W^{\pm}jj$@LO

- EW: $\sigma_{EW} \propto \alpha^6$



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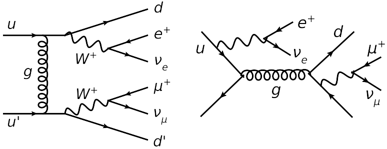
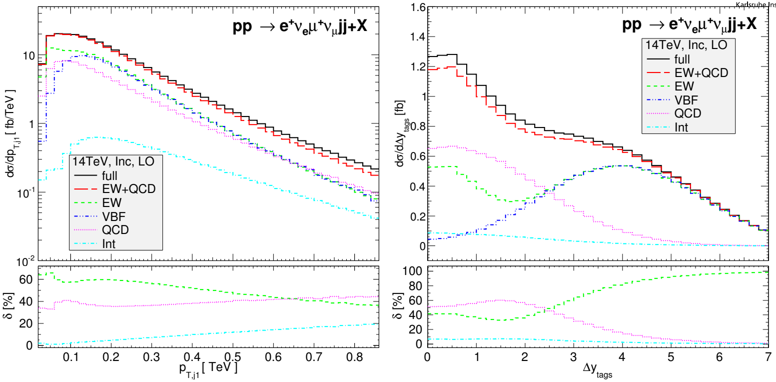


- Interference: $\sigma_{Int} \propto \alpha_s\alpha^5$, maximal for same-sign $W^{\pm}W^{\pm}jj$ due to the absence of gluon-induced processes and only left chiral quarks and leptons involve.

$W^{\pm}W^{\pm}jj$@LO: distributions

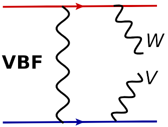


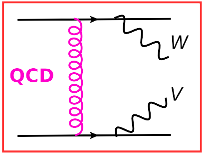
$$p_{T,j} > 20\text{ GeV},\quad |\eta_j| < 4.5,\quad R_{jj}^{anti-k_t} = 0.4,\quad R_{jl} > 0.4;$$

$$p_{T,l} > 20\text{ GeV},\quad |\eta_l| < 2.5,\quad R_{ll} > 0.4,\quad \not{p}_T > 30\text{ GeV}.$$

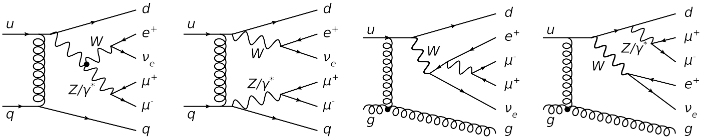
Dynamic scale: $\mu_F = \mu_R = \mu_0 = \left(\sum_{\text{partons}} p_{T,i} + \sum_i \sqrt{p_{T,W_i}^2 + m_{W_i}^2}\right)/2$,
$p_{T,W}$ (m_W) are reconstructed from leptons.
Interference effects: $\leq 15\%$ in relevant phase space region.

$pp \to VVjj$: QCD mechanism at NLO QCD





True *VVjj* processes

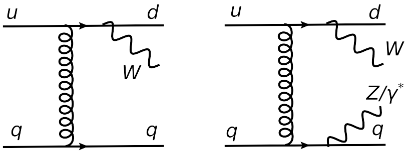


$+\ldots$

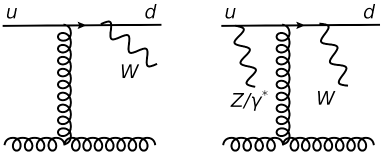
- Q: What is the NLO QCD correction to this process?
- A: Before answering this question, some classifications are needed.

LO: subprocesses

- 2-quark lines [4q]: $q_1 + q_2 \to q_3 + q_4 + (WV)$

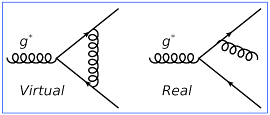


  - 76 subprocesses (2 generations).
  - 12 crossings.
- 1-quark line [2g]: $q_1 + q_2 \to g + g + (WV)$



  - 14 subprocesses (2 generations).
  - 7 crossings.
- 4 QCD gauge invariant groups: $4q(W)$, $4qWV$, $2g(W)$, $2gWV$.

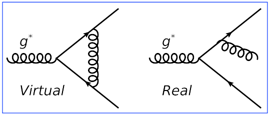
NLO calculation: theory

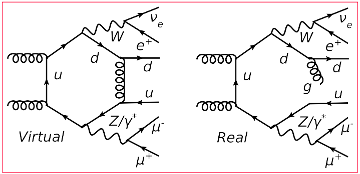


$$d\sigma_{NLO} = d\sigma^{\text{virt}}_{2\to N} + d\sigma^{\text{real}}_{2\to N+1}.$$

- Both terms are IR divergent. The sum is finite for IR-safe observables (e.g. jet distributions)

NLO calculation: theory vs. practice





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- Real: IR divergences can be separated using Catani-Seymour dipole subtraction method.
- Virtual: 1-loop amplitude is, unfortunately, much more complicated than tree-level one. Use Feynman-diagram and tensor reduction methods. The most difficult part.

Complexity overview

- LO: 4840 [*WZjj*, two generations]
- NLO real emission: 79784 [*WZjjj*, two generations]
- NLO virtual: 116896 (up to 6-point rank 5) [*WZjj*, two generations]
- Many subprocesses: most complicated *ZZjjj* (real emission) has 275 subprocesses (with b quarks)

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This calculation can be done with:

- good classifications: effective currents $V \to l_1 l_2$, $V \to l_1 l_2 l_3 l_4$, building blocks (hexlines, penlines, . . .), . . .
- use crossing symmetry (to a minimum extent to reduce computing time): subprocesses are not completely independent (diagrams share common parts).
- two independent calculations (Fact: manual calculations are buggy):
  - manual implementation using VBFNLO framework
  - more automated approach using HELAS/MadGraph, FeynArts, FormCalc
  - loop integrals: 2 different codes, in-house LoopInts and VBFNLO implementation.
  - tensor reduction: Passarino-Veltman, Denner-Dittmaier (2005).
- numerical instabilities in the virtual part: difficult $\rightsquigarrow$ gauge tests

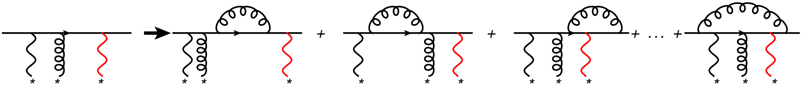
VBFNLO framework

- tree-level amplitudes, similar to MadGraph approach, but better optimization:

$$A =< f|i >, \quad |i >= \Gamma|i_1 >, < f_1|\Gamma =< f|,$$

where $|i >$ (or $< f|$) are just products of spinors, γ^μ, propagators and couplings. We have to identify the common (occurring in many diagrams and subprocesses) $|i >$, a part of a Feynman diagram, then calculate and save them at the beginning. This step is difficult to automatize $\rightsquigarrow$ manual implementation.

- one-loop amplitudes: built from universal (occurring in many processes, with generic couplings) building blocks, each is a group of Feynman diagrams with a fixed number and a fixed order of external particles. E.g. a penline:



- Effective currents: J^μ, ϵ^μ, p^μ $\rightsquigarrow$ efficient gauge tests.

FormCalc framework: semi-automated

- Adding leptonic decays directly does not work: code for $j_1 j_2 \to l_1 l_2 l_3 l_4 j_3 j_4$ cannot be generated (Mathematica+FORM takes forever).
- Tricks: generate Fortran code for semi-on-shell ($p^2 \neq M_V^2$) $VVjj$ and either replace $\epsilon^\mu \to J^\mu$ or cut the V propagators.
- Problems: Very long expression: use FormCalc6, for virtual *WZjj*, process ggqqVV: hex = 205M, pen = 84M, rest = 26M, tensor reduction not counted.
- Takes a lot of time to compile, code is slow, divide into small pieces: $\leadsto$ only useful to check at amplitude level.
- Bottom line: automated tools is faster in generating the amplitude, but much slower in providing cross section.

Numerical instabilities: gauge test

Passarino-Veltman: $D_{ijkl} = N/\det(G)^4$
$\rightsquigarrow$ small Gram-det problem.

$$\mathcal{B}^{\mathcal{N}} = T^N_\mu \epsilon^\mu(k),\ \epsilon^\mu \to k^\mu,$$

$$\frac{1}{\not{q}+\not{k}}\not{k}\frac{1}{\not{q}} = \frac{1}{\not{q}} - \frac{1}{\not{q}+\not{k}},$$

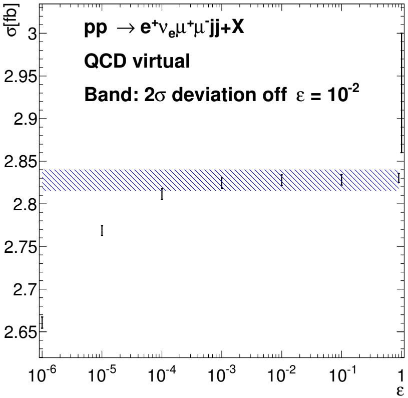
$$T^N_\mu k^\mu - \mathcal{A}^{N-1} = 0,$$

$$Q = 1 - \frac{\mathcal{A}^{N-1}}{T^N_\mu k^\mu},$$

if $Q < \varepsilon$: accept the point.

if $Q > \varepsilon$: use quadruple precision,

$\rightsquigarrow$ $Q < \varepsilon$? accept or discard points.



Computing time

With an Intel i5-3470 computer with one core and using the compiler Intel-ifort version 12.1.0, to get a statistical error of 1%:

- W^+W^+jj: 30 minutes.
- W^+Zjj: 2.5 hours.
- $W^+\gamma jj$: 3 hours. Importance: two integrals in two different regions on-shell $W^+ \to l^+\nu_l$ and on-shell $W^+ \to l^+\nu_l\gamma$. Two different Breit-Wigner mappings.
- $ZZjj$: 3.5 hours.

VVjj setup: cuts

- partons $\to$ jets: anti-k_t algorithm with a cone radius of $R = 0.4$.
- Inclusive cuts:

$$
\begin{aligned}
&p_{T,j} > 20\text{ GeV}, \quad |y_j| < 4.5, \quad R_{jj}^{anti-k_t} = 0.4, \quad R_{jl} > 0.4;\\
&p_{T,l} > 20\text{ GeV}, \quad |y_l| < 2.5, \quad R_{ll} > 0.4, \quad M_{l^+l^-} > 15\text{ GeV};\\
&p_{T,\gamma} > 30\text{ GeV}, \quad |y_\gamma| < 2.5, \quad R_{l\gamma} > 0.4, \quad R_{j\gamma} > 0.7, \quad \not{p}_T > 30\text{ GeV}.
\end{aligned}
$$

- Final-sate real photon: Frixione smooth cone isolation cut. Events are accepted if

$$\sum_{i\in\text{partons}} p_{T,i}\theta(R - R_{\gamma i}) \le p_{T,\gamma}\frac{1-\cos R}{1-\cos\delta_0} \quad \forall R < \delta_0$$

with $\delta_0 = 0.7 \leadsto$ events with a soft gluon are accepted (IR safety).

VVjj setup: scale choice

$$\mu_{\mathrm{HT}} = \left(\sum_{\text{partons}} p_{T,i} + \sum_{i=1}^{2} E_{T,V_i} \right) /2 \quad (1)$$

$$\mu'_{\mathrm{HT}} = \left(\sum_{\text{jet}} p_{T,i} e^{|y_i - y_{12}|} + \sum_{i=1}^{2} E_{T,V_i} \right) /2 \quad (2)$$

$$\mu_{\mathrm{ET}} = \left[E_T(jj) + E_T(VV) \right] /2 \quad (3)$$

with $y_{12} = (y_1 + y_2)/2$ and $E_T(jj) = (m_{jj}^2 + p_{jj}^2)^{1/2}$, and

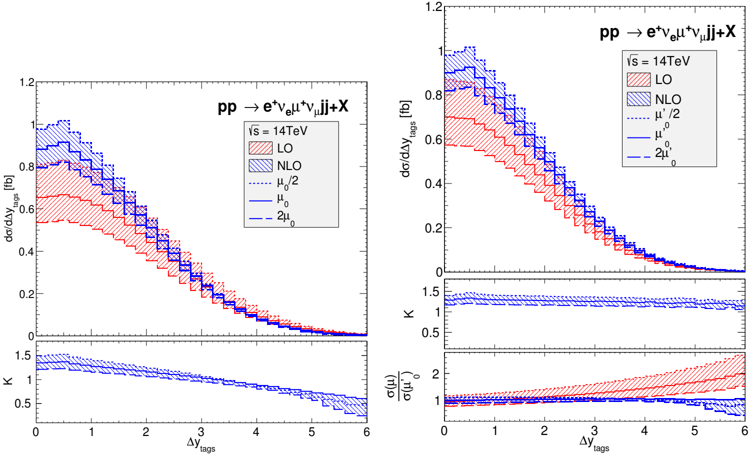
$$m_{jj}^2 \approx 2 p_{T,j_1} p_{T,j_2} \left[\cosh(\Delta y_{jj}) - \cos(\Delta\phi_{jj}) \right]$$

Remarks:

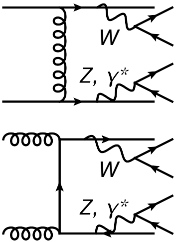
- For large Δy_{jj}, then $\sum p_T \ll m_{jj} \rightsquigarrow \mu_{\mathrm{HT}}$ too small.
- μ'_{HT} and μ_{ET} interpolate between $\sum p_T$ and m_{jj}, for small and large Δy_{jj}, respectively.

Δy_{jj} distribution: scale choice

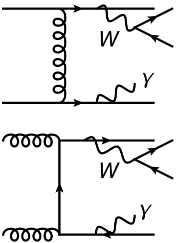
$$\mu_0 = \left(\sum_{\text{partons}} p_{T,i} + \sum_i E_{T,w_i}\right)/2; \qquad \mu_0' = \left(\sum_{\text{jets}} p_{T,i} e^{|y_i - y_{12}|} + \sum_i E_{T,w_i}\right)/2$$



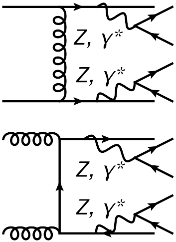
Overview



W^+Z
Leptonic decays
Off-shell effects
Virtual photon



$W^+\gamma$
Leptonic decays
Off-shell effects
Real photon



ZZ
Leptonic decays
Off-shell effects
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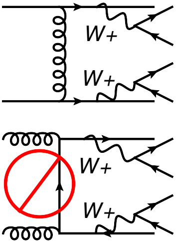
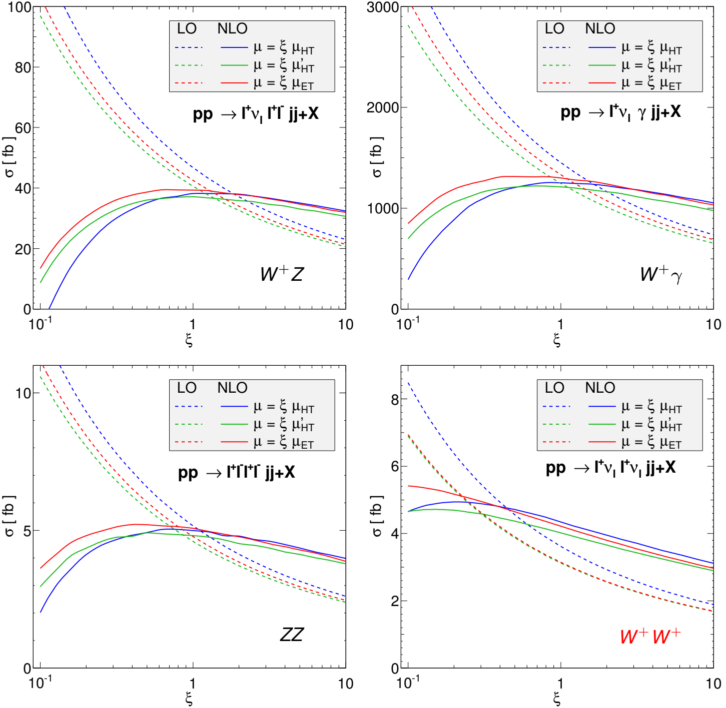
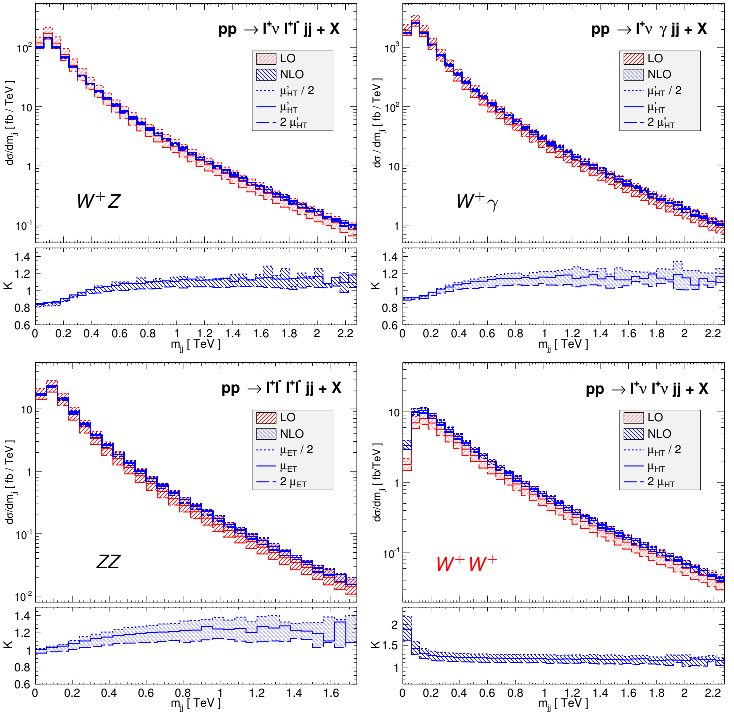


W^+W^+
Leptonic decays
Off-shell effects

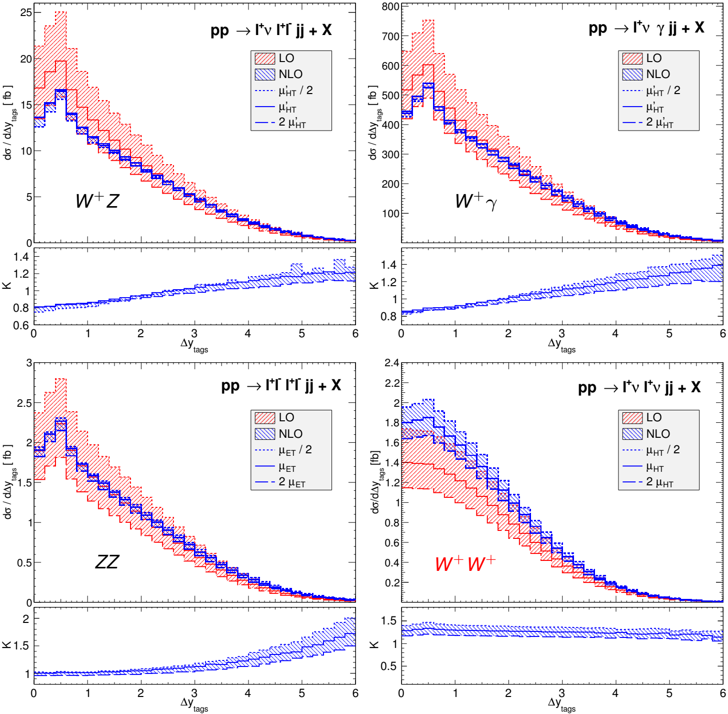
Scale dependence



m_{jj} distributions



Δy_{jj} distributions

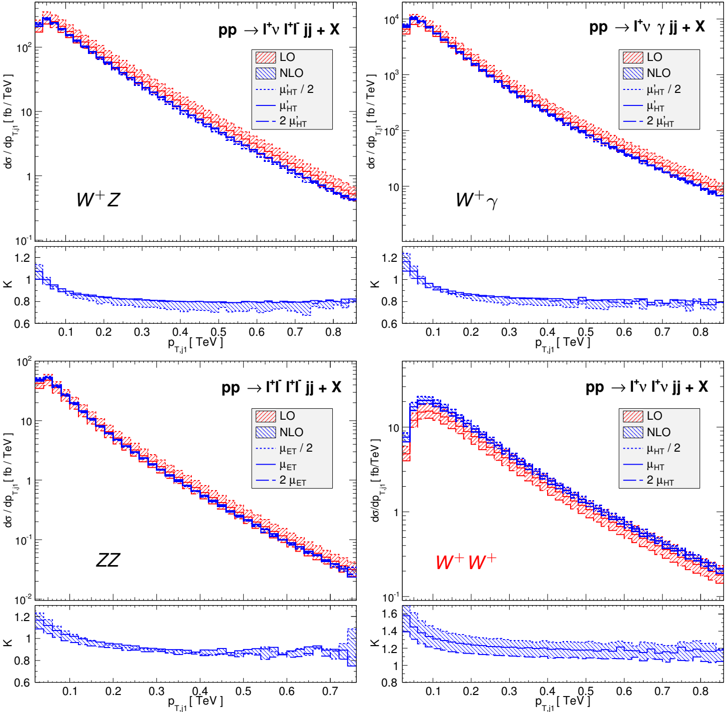


Summary

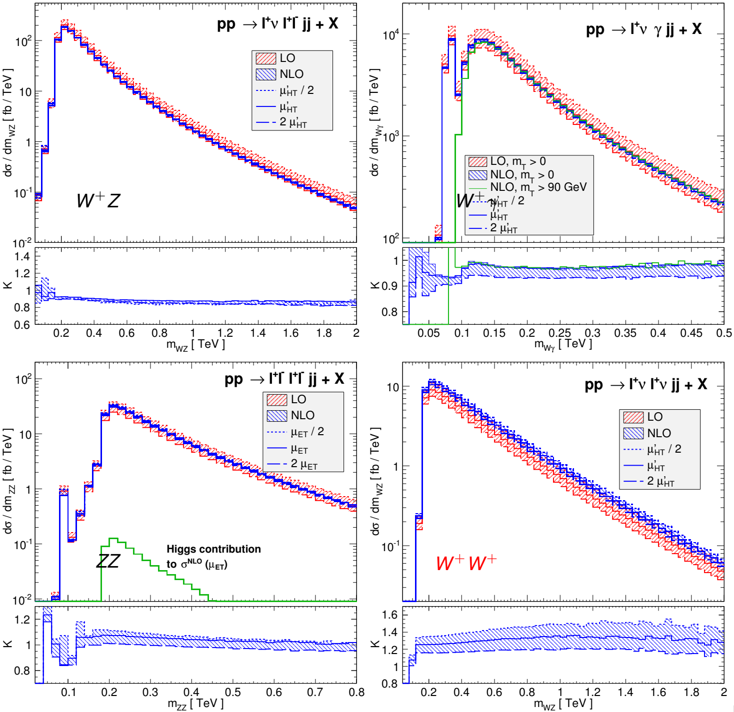
- *VVjj* is a special class of processes: sensitive to $VV \to VV$ scatterings, quartic gauge couplings, background for new physics searches, ...
- Two mechanisms: EW (VBF), QCD, interference effects are small (at most 15% for W^+W^+jj, expected $\leq 5\%$ for the others).
- NLO QCD corrections for all *VVjj* are under control.
- A good scale choice has to take m_{jj} into account, p_T alone is not enough.
- Virtual amplitude: difficult part, gauge tests are good to deal with numerical instabilities.
- The program VBFNLO 2.7.0: includes $WZjj$, $W\gamma jj$, $W^\pm W^\pm jj$. $ZZjj$ will be in the next release (or upon request).

Thank you!

p_{T,j_1} distributions

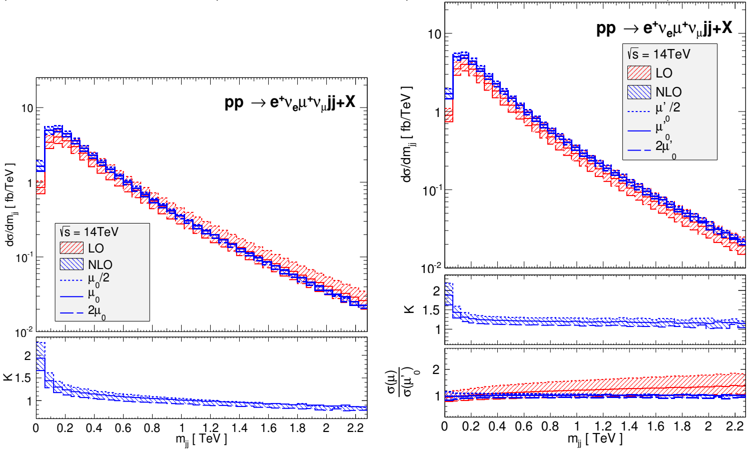


m_{VV} distributions



m_{jj} distribution: scale choice

$\mu_0 = \left(\sum_{\text{partons}} p_{T,i} + \sum_i E_{T,w_i}\right)/2;$ $\mu'_0 = \left(\sum_{\text{jets}} p_{T,i} e^{|y_i - y_{12}|} + \sum_i E_{T,w_i}\right)/2$



But, large K factor at small m_{jj} always there: new $q_{final} \to j_1 j_2$ configuration at NLO.

Dipole subtraction method

$$\int_{N+1} d\sigma^{\text{real}}_{N+1}(p) J^{N+1}(p) = \int_{N+1} \left[\left(d\sigma^{\text{real}}_{N+1}(p) J^{N+1}(p) - \sum_{i,j} S^N_{ij}(\tilde{p}_{ij}) J^N_{ij}(\tilde{p}_{ij}) \right) \right] + \underbrace{\int_{N+1} \sum_{i,j} S^N_{ij}(\tilde{p}_{ij}) J^N_{ij}(\tilde{p}_{ij})}_{\text{PK+I}} \quad (4)$$

$$\text{PK} = \int_0^1 dx \int_N \sum_{j \neq a} S^N_{aj}(x,p) J^N_a(x,p) + (a \leftrightarrow b) \quad (5)$$

$$\text{I} = \int_N \sum_{i,j} S^N_{ij}(p) J^N(p). \quad (6)$$

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The Jet function: a cut, a histogram, PDF(Q), $\alpha_s(Q)$, ...

- IR safety: $J^N \to J^{N+1}$ in the IR singular limits.
- An easy mistake: in Eq. (4), set $\alpha_s^N(\tilde{Q}) = \alpha_s^{N+1}(Q)$: the result is finite, BUT wrong (almost correct) because the integrated part $\neq$ the subtraction part.
- If we do $J^N = J^{N+1}$ in Eq. (4), then PK term will get more complicated. [arXiv: 0802.1405]

Tree-level, virtual and real matching

- Partonic level:

$$\begin{aligned} d\sigma^{\mathrm{I}}_{\mathrm{soft}} + d\sigma^{\mathrm{virt}}_{\mathrm{soft}} &= 0, \\ d\sigma^{\mathrm{I}}_{\mathrm{coll}} + d\sigma^{\mathrm{virt}}_{\mathrm{coll}} &= 0, \\ d\sigma^{\overline{PK}}_{\mathrm{coll}} &\neq 0. \end{aligned}$$

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- Hadronic level:

$$d\sigma^{\mathrm{NLO}} = d\sigma^{\mathrm{tree}} + d\sigma^{\mathrm{virt}} + d\sigma^{\mathrm{real}},$$
$$\mathrm{PDF}^{\mathrm{NLO}} \otimes d\sigma^{\mathrm{NLO}} = \int dx_1 \int dx_2 f_a(x_1, Q) f_b(x_2, Q) d\sigma^{\mathrm{NLO}} + \delta_{\mathrm{PDF}}(d\sigma^{\mathrm{tree}}, 1/\epsilon),$$
$$d\sigma^{\mathrm{PK}} \equiv d\sigma^{\overline{PK}}(1/\epsilon) + \delta_{\mathrm{PDF}}(d\sigma^{\mathrm{tree}}, 1/\epsilon) : \quad \text{finite}$$

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- Default matching: PDF and $d\sigma^{\mathrm{real}}$ are defined in the conventional dimensional-regularization scheme (CDR), AND $\overline{MS}$ scheme.

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- Default matching: PDF and $d\sigma^{\text{real}}$ are defined in the conventional dimensional-regularization scheme (CDR), AND $\overline{MS}$ scheme.
- Virtual: use 't Hooft-Veltman (HV) scheme (external momenta in 4D)

$$d\sigma^{\text{virt}}_{\text{HV}} = d\sigma^{\text{virt}}_{\text{CDR}}, \quad \alpha_s^{\text{HV}} = \alpha_s^{\text{CDR}}.$$

- Dimensional regularization scheme (DRS) independence [Catani, Seymour, Trócsányi 1997]:

$$d\sigma^{\text{tree}} + d\sigma^{\text{virt}} + d\sigma^{\text{I}} : \notin \text{DRS at partonic level!}$$

- 4 flavors (no b/t) or 5 flavors (b and t loop): α_s, PDF, tree, virtual, real.

Subtraction term: implementation for $2 \to N+1$

$$\sigma^{N+1} = \sum_{\text{subproc}} \int_{N+1} \left[d\sigma^{\text{real}}_{N+1}(p) - \sum_{i,j,k} S^{N}_{ijk}(\tilde{p}_{ijk}) \right]$$

- Do for one subprocess, then use crossing: 27 crossings, caching (M. Kerner).

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- Universal kinematic mapping $M(i,j;k)$: emitter (p_i), unresolved (p_j), spectator (p_k) with $i,k \in \{1, N+1\}$; $j \in \{3, N+1\}$.
  - final-final ($i,k > 2$): $M_{ff}(i,j;k) = M_{ff}(j,i;k) \neq M_{ff}(k,j;i)$. $\rightsquigarrow N+1$
  - final-initial ($i > 2, a \leq 2$): $M_{fi}(i,j;a) = M_{fi}(j,i;a)$. $\rightsquigarrow N(N+1)$
  - initial-final ($a \leq 2, k > 2$): $M_{if}(a,j;k) = M_{fi}(k,j;a)$. $\rightsquigarrow 0$
  - initial-initial ($a \leq 2, b \leq 2$): $M_{ii}(a,j;b) \neq M_{ii}(b,j;a)$. $\rightsquigarrow 2(N+1)$
  - Number of independent mappings: $(N+1)(N+3)$.

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  - Number of independent mappings: $(N+1)(N+3)$.
- Each QCD dipole: kinematics, splitting (vertex, propagator), Born amplitude matrix (mapped kinematics). A generic Fortran code!
- Input: Born amplitude matrix (helicity, color), color-correlated matrices. Sum over subprocesses is optimized here.
- Check: singular limits (necessary BUT not enough).

PK term: implementation

$$\sigma_{\rm PK}^{N} = \int_0^1 dy \int_0^1 dx_1 \int_0^1 dx_2 f_a(x_1) f_b(x_2)$$

$$\times \left\{ \sum_{a_1} \int_N [{\rm PK}^{aa_1} d\sigma_{a_1 b}(y p_a, p_b)] + \sum_{b_1} \int_N [{\rm PK}^{bb_1} d\sigma_{a b_1}(p_a, y p_b)] \right\},$$

$${\rm PK}(y) = A(y) + \delta(1-y)B(y) + [C(y)]_+ D(y),$$

$$\int_0^1 [C(y)]_+ E(y) dy \equiv \int_0^1 C(y)[E(y) - E(1)] dy, \quad E(y) = D(y) d\sigma(y).$$

- Straight-forward coding: calculate Born amplitudes twice $d\sigma(y)$ and $d\sigma(1)$.

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- Straight-forward coding: calculate Born amplitudes twice $d\sigma(y)$ and $d\sigma(1)$.
- Trick: $z_1 = x_1 y, z_2 = x_2$ if a is the emitter; $z_1 = x_1, z_2 = x_2 y$ if b is the emitter:

$$
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&\times \left\{ \sum_{a_1} \int_N d\sigma_{a_1 b}(z_1 z_2)[\mathrm{PK}^{aa_1} f_a(z_1/y) f_b(z_2) \theta(y - z_1)] \right. \\
&+ \left. \sum_{b_1} \int_N d\sigma_{ab_1}(z_1 z_2)[\mathrm{PK}^{bb_1} f_a(z_1) f_b(z_2/y) \theta(y - z_2)] \right\}.
\end{aligned}
$$

- A generic Fortran code. Input: $\sum_{\rm subproc} |A_0|^2(c_1, c_2)$ used for subtraction term.

I term: implementation

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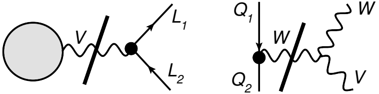
$$\sigma_{\mathrm{I}}^{N} = \int_N \sum_{i\neq j} \mathrm{I}_{ij} \otimes d\sigma_{\text{tree}}.$$

- A generic Fortran code. Input: color-correlated squared amplitudes.
- Good check for PKI: α parameter [Nagy and Trocsanyi, M. Kerner in VBFNLO].

Virtual amplitude

- Include V decays:

$$\epsilon^{\mu}(k,\lambda) \to J^{\mu}_{\text{eff}}/(k^2 - M_V^2 + iM_V\Gamma_V), \quad \text{Or}$$



$$g^{\mu\nu} = -\sum_{\lambda=-1,0,1} \epsilon^{\mu}(k,\lambda)\epsilon^{*\nu}(k,\lambda) + \underbrace{\frac{k^{\mu}k^{\nu}}{k^2}}_{=0},$$

$$k^{\mu}\bar{F}_1\gamma_{\mu}(a + b\gamma_5)F_2 = 0, \quad \text{since}: m_1 = m_2 = 0 \;\; \text{(no Goldstones).}$$

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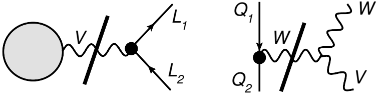

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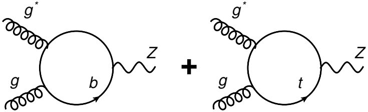
- Fermion loop with γ_5: anomaly free (need both b and t)



$\rightsquigarrow$ 2.5 generations not OK (we cannot decouple the top quark).

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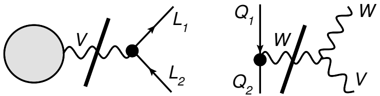

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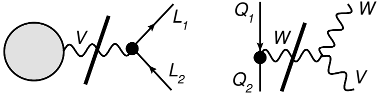

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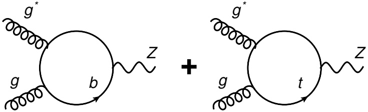

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