Antenna Subtraction

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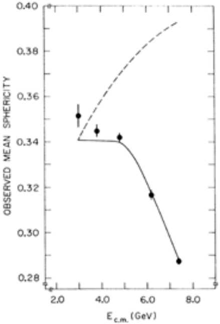
Outline

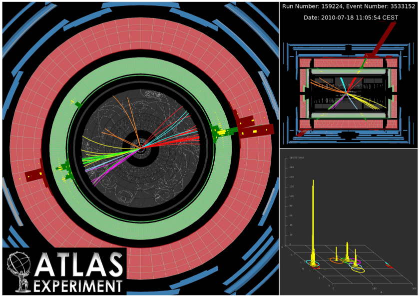
- Introductory remarks
  - motivation for jet physics
  - higher order calculations
  - rival methods, relative advantages and disadvantages
- Antenna Subtraction:
  - what is an antenna?
  - how do you use one?
- Colourful Antenna Subtraction:
  - why colour explicit?
  - antenna dipoles
  - spin structure
- NNLO Dijets
  - quark-gluon channel
  - identity changing issues
  - update on results

Jets, past and present

Jets are *the only available* high energy experimental QCD object



[Phys. Rev. Lett. 35: 1609 (1975)]



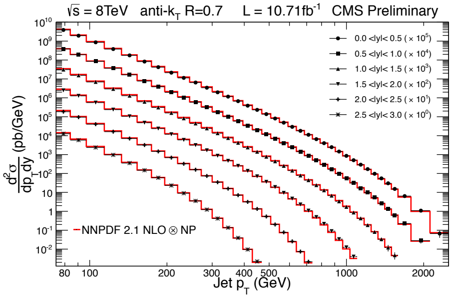
$m_{jj} \sim 2.55\text{TeV},\ p_{t_1} = 420\text{GeV},\ p_{t_2} = 320\text{GeV}$

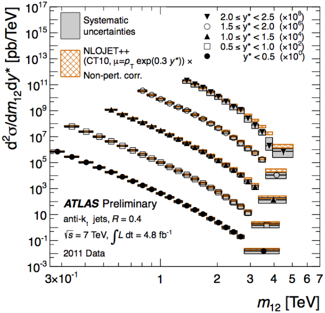
Many process of interest involve at least one jet in the final state:

- $pp \to jj$
- $pp \to H + j$
- $pp \to V + j$
- $ep \to (2+1)j$

Cross sections accurately measured and presented in differential form, e.g.

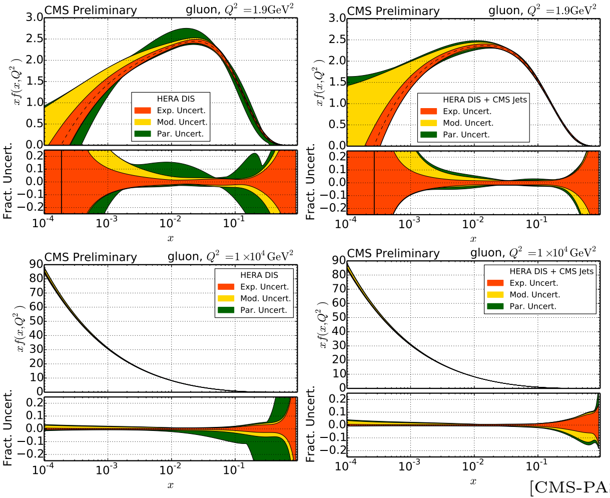
- single jet inclusive w.r.t p_T and $|y|$
- exclusive dijet w.r.t m_{jj} and y^*





Uses of jet data - PDFs

Single jet inclusive x-sec, constrain PDFs, in particular the gluon at large x

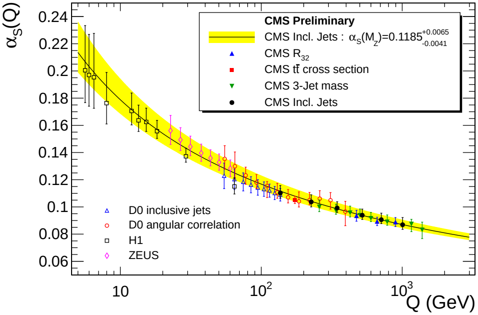


[CMS-PAS-SMP-12-028]

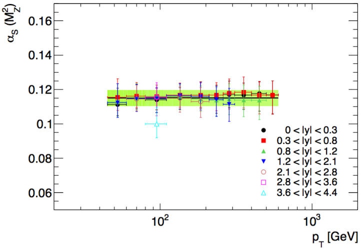
Uses of jet data - α_s

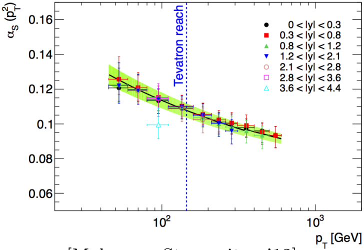
Can use single jet inclusive x-sec to fit:

- $\alpha_s(M_Z)$
- running coupling



[CMS-PAS-SMP-12-028]

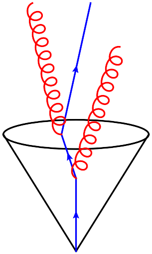




[Malaescu, Starovoitov '12]

Higher order corrections

- Large reduction in theoretical scale uncertainty
  - needed for a precise determination of α_s
- Improved perturbative convergence
- NNLO coefficient needed to fit NNLO PDFs
- Corrections change the normalization, but also the shape
- Jet algorithm much more realistic at NNLO
  - parton shower inside the jet using pQCD
  - additional jets from branching

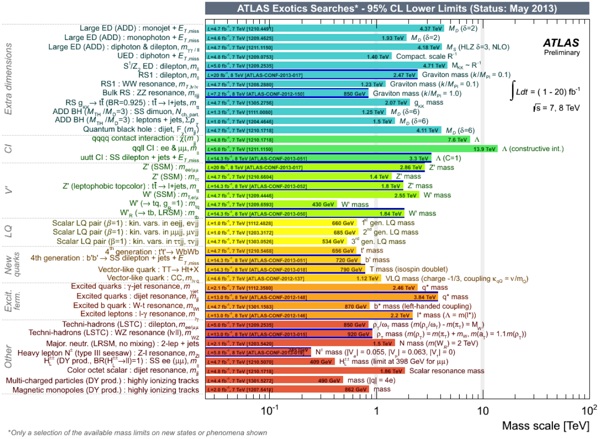


- Initial-state radiation
  - more realistic final-state p_T, reduce the need for intrinsic p_T
- logarithmic corrections in pert. theory vs NP power corrections

We need NNLO pQCD corrections for a realistic simulation of the events

For the BSM dreamers...

The Standard Model is working well. . . maybe too well



For the BSM dreamers...

The Standard Model is working well... maybe too well

- Scenario 1: *"that's all folks!"*
  - no new physical states below Λ_{GUT}, Λ_R, Λ_{PC}, Λ_{Pl}
  - finely tuned Standard Model?
  - more radical approaches, e.g. non-commutative geometry
- Scenario 2:
  - new physics is there, but hiding
  - compressed SUSY spectrum
  - BSM mimicking SM
  - just out of reach of LHC... VLHC?
  - precise understanding of SM boosts our ability to resolve BSM

The NNLO marketplace

In recent years many new tools developed for NNLO

- all have advantages and disadvantages

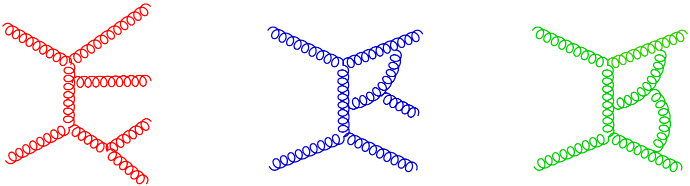
	analytic	FS colour	IS colour	local
antenna subtraction	✓	✓	✓	✗
STRIPPER	✗	✓	✓	✓
q_T subtraction	✓	✗	✓	✓
reverse unitarity	✓	✗	✓	-
full ME subtraction	✗	✓	✗	✓

Antenna subtraction is the only method for computing cross sections with:

- hadronic initial-states
- jets in the final-state (especially more than one jet)
- analytic pole cancellation

Subtraction at NNLO

$$
\begin{aligned}
d\hat{\sigma}_{ab,NNLO} &= \int_{\Phi_{m+2}} d\hat{\sigma}^{RR}_{ab,NNLO} \\
&+ \int_{\Phi_{m+1}} \left[d\hat{\sigma}^{RV}_{ab,NNLO} + d\hat{\sigma}^{MF,1}_{ab,NNLO} \right] \\
&+ \int_{\Phi_{m}} \left[d\hat{\sigma}^{VV}_{ab,NNLO} + d\hat{\sigma}^{MF,2}_{ab,NNLO} \right]
\end{aligned}
$$



Subtraction at NNLO

$$\begin{aligned} \mathrm{d}\hat{\sigma}_{ab,NNLO} &= \int_{\Phi_{m+2}} \left[\mathrm{d}\hat{\sigma}^{RR}_{ab,NNLO} - \mathrm{d}\hat{\sigma}^{S}_{ab,NNLO}\right] \\ &+ \int_{\Phi_{m+1}} \left[\mathrm{d}\hat{\sigma}^{RV}_{ab,NNLO} - \mathrm{d}\hat{\sigma}^{T}_{ab,NNLO}\right] \\ &+ \int_{\Phi_{m}} \left[\mathrm{d}\hat{\sigma}^{VV}_{ab,NNLO} - \mathrm{d}\hat{\sigma}^{U}_{ab,NNLO}\right] \end{aligned}$$

$$\begin{aligned} \mathrm{d}\hat{\sigma}^{T}_{ab,NNLO} &= -\int_1 \mathrm{d}\hat{\sigma}^{S}_{ab,NNLO} + \mathrm{d}\hat{\sigma}^{V,S}_{ab,NNLO} - \mathrm{d}\hat{\sigma}^{MF,1}_{ab,NNLO} \\ \mathrm{d}\hat{\sigma}^{U}_{ab,NNLO} &= -\int_2 \mathrm{d}\hat{\sigma}^{S}_{ab,NNLO} - \int_1 \mathrm{d}\hat{\sigma}^{V,S}_{ab,NNLO} - \mathrm{d}\hat{\sigma}^{MF,2}_{ab,NNLO} \end{aligned}$$

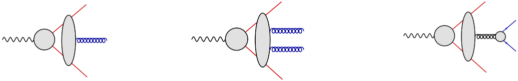
What is an antenna?

Constructed from physical matrix elements

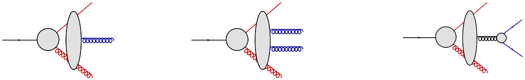
$$X_3^0(i,j,k) \sim \frac{|\mathcal{M}_3^0(i,j,k)|^2}{|\mathcal{M}_2^0(I,K)|^2}, \qquad X_4^0(i,j,k,l) \sim \frac{|\mathcal{M}_4^0(i,j,k,l)|^2}{|\mathcal{M}_2^0(I,L)|^2}$$

Three main types:

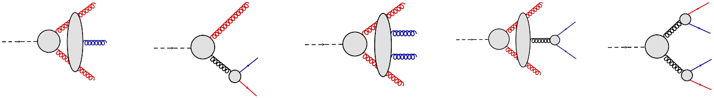
- Quark-antiquark. Derived from the process $\gamma^* \to q\bar{q} + \cdots$



- Quark-gluon. Derived from the process $\tilde{\chi}^0 \to \tilde{g}g + \cdots$

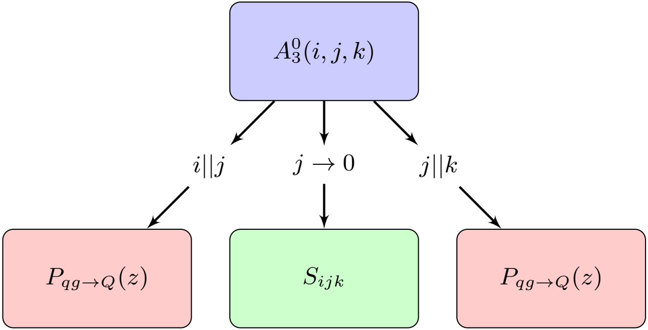


- Gluon-gluon. Derived from the process $H \to gg + \cdots$



How are they useful?

- smoothly interpolates many unresolved limits



- analytically integrable. . . and integrated

Extending to NNLO

IR factorization depends on colour connection of partons,

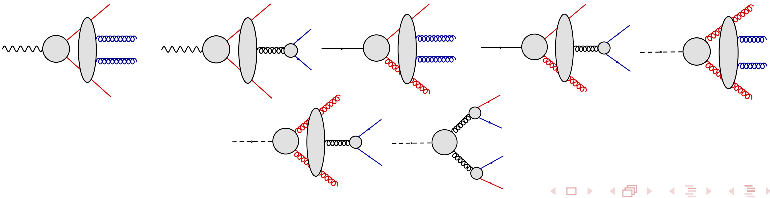
$$|\mathcal{M}_n^0(\cdots, a, i, j, k, l, b\cdots)|^2$$

- i, l "colour disconnected"
- i, k and j, l "almost colour connected"
- j, k "colour connected"

For disconnected and almost colour connected singularities

$$X_3^0(a, i, j)\ X_3^0(J, k, l)$$

For colour connected singularities we need $X_4^0(i, j, k, l)$



Antenna Subtraction Toolbox

Many tools needed for implementation:

- final-final phase space mappings [Kosower '03]
- FF X_3^0, X_4^0, X_3^1 antennae [Gehrmann-De Ridder, Gehrmann, Glover, '04, '05]
- integrated FF antennae [Gehrmann-De Ridder, Gehrmann, Glover, '05]

$\Rightarrow$ $e^+e^- \rightarrow$ 3 jets at NNLO [Gehrmann-De Ridder, Gehrmann, Glover, Heinrich, '07, Weinzierl '08]

Since then, extended for hadronic initial-states:

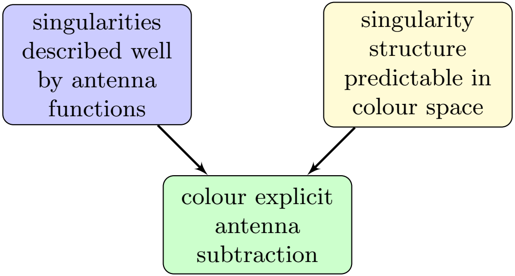
- initial-final + initial-initial mappings [Daleo, Gehrmann, Maître, '07]
- integrated IF X_3^1, X_4^0 [Daleo, Gehrmann-De Ridder, Gehrmann, Luisoni, '10]
- integrated II X_4^0 [Boughezal, Gehrmann-De Ridder, Ritzmann, '11. Gehrmann, Ritzmann '12]
- integrated II X_3^1 [Gehrmann, Monni, '11]

All tools exist for hadron-hadron scattering

[Glover, Pires, '10. Gehrmann De-Ridder, Glover, Pires, '12. Gehrmann De-Ridder, Gehrmann, Glover, Pires ,'13. JC, Glover, Wells, '13. JC, Gehrmann De-Ridder, Glover, Pires, '14.]

Colour explicit antenna subtraction

- Take the best bits from usual method...the antennae and PS mappings
- Match onto the predictable colour space virtual singularity structure



- Resulting method needs no new integrals and applies to arbitrary processes

1-loop singularities - Catani recap

Move into colour space with set of abstract basis vectors $|\boldsymbol{c}\rangle$

$$\mathcal{M}_n = \langle \boldsymbol{c}|\mathcal{M}_n\rangle \quad \& \quad |\mathcal{M}_n|^2 = \langle \mathcal{M}_n|\mathcal{M}_n\rangle$$

One-loop amplitude pole structure governed by colour dipole operator,

$$|\mathcal{M}_n^1\rangle \quad = \quad \boldsymbol{I}^{(1)}(\epsilon)|\mathcal{M}_n^0\rangle + \mathcal{O}(\epsilon^0)$$

$$\boldsymbol{I}^{(1)}(\epsilon) \quad = \quad \sum_{\text{pairs}(i,j)} \boldsymbol{\mathcal{I}}_{ij}^{(1)}(\epsilon)\ (\boldsymbol{T}_i \cdot \boldsymbol{T}_j)$$

One-loop matrix element pole structure given by,

$$2\text{Re}\Big[\langle \mathcal{M}_n^0|\mathcal{M}_n^1\rangle\Big] \quad = \quad \sum_{(i,j)} 2\text{Re}\Big[\boldsymbol{\mathcal{I}}_{ij}^{(1)}(\epsilon)\Big]\langle \mathcal{M}_n^0|(\boldsymbol{T}_i \cdot \boldsymbol{T}_j)|\mathcal{M}_n^0\rangle + \mathcal{O}(\epsilon^0)$$

Integrated antenna dipoles

Integrated antennae contain Catani pole coefficients:

- set up correspondence

$$\boldsymbol{\mathcal{I}}_{ij}^{(1)}(s_{ij}) \Leftrightarrow \boldsymbol{\mathcal{J}}_2^{(1)}(i,j)$$

- define an antenna insertion operator

$$\boldsymbol{\mathcal{J}}^{(1)}(\epsilon) = \sum_{(i,j)} \boldsymbol{\mathcal{J}}_2^{(1)}(i,j)\ (\boldsymbol{T}_i \cdot \boldsymbol{T}_j)$$

- virtual poles written systematically in terms of integrated antennae

$$2\mathrm{Re}\left[\langle \mathcal{M}_n^0 | \mathcal{M}_n^1 \rangle\right] = 2\sum_{(i,j)} \boldsymbol{\mathcal{J}}_2^{(1)}(i,j) \langle \mathcal{M}_n^0 | (\boldsymbol{T}_i \cdot \boldsymbol{T}_j) | \mathcal{M}_n^0 \rangle + \mathcal{O}(\epsilon^0)$$

- clear link between $\boldsymbol{\mathcal{J}}_2^{(1)}(i,j) \sim \mathcal{X}_3^0(s_{ij})$ and X_3^0

Example: $e^+e^- \to 3j$ @ NLO…colour stripped approach

The $q\bar{q}gg$ real correction to $e^+e^- \to 3j$.

$$\mathrm{d}\hat{\sigma}^{R}_{NLO} \sim \int_{\Phi_4} \sum_{(i,j)} \left[M_4^0(1_q, i_g, j_g, 2_{\bar{q}}) - \frac{1}{2N^2} \widetilde{M}_4^0(1_q, i_g, j_g, 2_{\bar{q}}) \right] J_3^{(4)}$$

Example: $e^+e^- \to 3j$ @ NLO...colour stripped approach

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$$\mathrm{d}\hat{\sigma}^{S}_{NLO} \sim \int_{\Phi_4} \sum_{(i,j)} \left[d_3^0(1,i,j) M_3^0((1i)_q, (ij)_g, 2_{\bar{q}}) + d_3^0(2,j,i) M_3^0(1_q, (ij)_g, (2j)_{\bar{q}}) - \frac{1}{N^2} A_3^0(1,i,2) M_3^0((1i)_q, j_g, (2i)_{\bar{q}}) \right] J_3^{(3)}$$

Example: $e^+e^- \to 3j$ @ NLO... colour stripped approach

The $q\bar{q}gg$ real correction to $e^+e^- \to 3j$.

$$\mathrm{d}\hat{\sigma}^R_{NLO} \sim \int_{\Phi_4} \sum_{(i,j)} \left[M_4^0(1_q, i_g, j_g, 2_{\bar{q}}) - \frac{1}{2N^2} \widetilde{M}_4^0(1_q, i_g, j_g, 2_{\bar{q}}) \right] J_3^{(4)}$$

$$\mathrm{d}\hat{\sigma}^S_{NLO} \sim \int_{\Phi_4} \sum_{(i,j)} \left[d_3^0(1,i,j) M_3^0((1i)_q, (ij)_g, 2_{\bar{q}}) + d_3^0(2,j,i) M_3^0(1_q, (ij)_g, (2j)_{\bar{q}}) \right.$$
$$\left. - \frac{1}{N^2} A_3^0(1,i,2) M_3^0((1i)_q, j_g, (2i)_{\bar{q}}) \right] J_3^{(3)}$$

$$\mathrm{d}\hat{\sigma}^T_{NLO} \sim \int_{\Phi_3} \left[\underbrace{\frac{1}{2}\mathcal{D}_3^0(s_{13}) + \frac{1}{2}\mathcal{D}_3^0(s_{23})}_{2I_{qg}^{(1)}(s_{13})+2I_{qg}^{(1)}(s_{23})} - \frac{1}{N^2} \underbrace{\mathcal{A}_3^0(s_{12})}_{2I_{q\bar{q}}^{(1)}(s_{12})} \right] M_3^0(1_q, 3_g, 2_{\bar{q}}) J_3^{(3)}$$

Example: $e^+e^- \to 3j$ @ NLO…colour stripped approach

The $q\bar{q}gg$ real correction to $e^+e^- \to 3j$.

$$d\hat{\sigma}^R_{NLO} \sim \int_{\Phi_4} \sum_{(i,j)} \left[M_4^0(1_q, i_g, j_g, 2_{\bar{q}}) - \frac{1}{2N^2} \widetilde{M}_4^0(1_q, i_g, j_g, 2_{\bar{q}}) \right] J_3^{(4)}$$

$$d\hat{\sigma}^S_{NLO} \sim \int_{\Phi_4} \sum_{(i,j)} \left[d_3^0(1,i,j) M_3^0((1i)_q, (ij)_g, 2_{\bar{q}}) + d_3^0(2,j,i) M_3^0(1_q, (ij)_g, (2j)_{\bar{q}}) - \frac{1}{N^2} A_3^0(1,i,2) M_3^0((1i)_q, j_g, (2i)_{\bar{q}}) \right] J_3^{(3)}$$

$$d\hat{\sigma}^T_{NLO} \sim \int_{\Phi_3} \left[\underbrace{\frac{1}{2}\mathcal{D}_3^0(s_{13}) + \frac{1}{2}\mathcal{D}_3^0(s_{23})}_{2I_{qg}^{(1)}(s_{13}) + 2I_{qg}^{(1)}(s_{23})} - \frac{1}{N^2} \underbrace{\mathcal{A}_3^0(s_{12})}_{2I_{q\bar{q}}^{(1)}(s_{12})} \right] M_3^0(1_q, 3_g, 2_{\bar{q}}) J_3^{(3)}$$

$$d\hat{\sigma}^V_{NLO} \sim \int_{\Phi_3} \left[2I_{qg}^{(1)}(s_{13}) + 2I_{qg}^{(1)}(s_{23}) - \frac{1}{N^2} 2I_{q\bar{q}}^{(1)}(s_{12}) \right] M_3^0(1_q, 3_g, 2_{\bar{q}}) J_3^{(3)} + \mathcal{O}(\epsilon^0)$$

Example: $e^+e^- \to 3j$ @ NLO…colour explicit approach

For three partons $q_1,\ \bar{q}_2,\ g_3$,

$$\boldsymbol{T}_1 \cdot \boldsymbol{T}_3 = \boldsymbol{T}_2 \cdot \boldsymbol{T}_3 = -\frac{N}{2}\,\mathbf{1}\,,\ \boldsymbol{T}_1 \cdot \boldsymbol{T}_2 = \frac{1}{2N}\,\mathbf{1}$$

$$\boldsymbol{\mathcal{J}}^{(1)}(\epsilon) = \boldsymbol{\mathcal{J}}_2^{(1)}(1,3)\,(\boldsymbol{T}_1 \cdot \boldsymbol{T}_3) + \boldsymbol{\mathcal{J}}_2^{(1)}(2,3)\,(\boldsymbol{T}_2 \cdot \boldsymbol{T}_3) + \boldsymbol{\mathcal{J}}_2^{(1)}(1,2)\,(\boldsymbol{T}_1 \cdot \boldsymbol{T}_2)$$

Construct the virtual subtraction term,

$$\mathrm{d}\hat{\sigma}^T_{NLO} \sim \int_{\Phi_3} \langle \mathcal{M}_3^0 | \boldsymbol{\mathcal{J}}^{(1)}(\epsilon) | \mathcal{M}_3^0 \rangle$$

$$\mathrm{d}\hat{\sigma}^T_{NLO} \sim \int_{\Phi_3} \Big[\underbrace{\boldsymbol{\mathcal{J}}_2^{(1)}(1,3)}_{\frac{1}{2}\mathcal{D}_3^0(s_{13})} + \underbrace{\boldsymbol{\mathcal{J}}_2^{(1)}(2,3)}_{\frac{1}{2}\mathcal{D}_3^0(s_{23})} - \frac{1}{N^2} \underbrace{\boldsymbol{\mathcal{J}}_2^{(1)}(1,2)}_{\mathcal{A}_3^0(s_{12})} \Big] \langle \mathcal{M}_3^0 | \mathcal{M}_3^0 \rangle$$

- same result as colour stripped approach

Unintegrated insertion operator,

$$\mathbb{X}_3^0 = \sum_{(i,j)\in(3,4)} \Big[X_3^0(1,i,j)\,(\boldsymbol{T}_{(1i)}\cdot\boldsymbol{T}_{(ij)}) + X_3^0(2,i,j)\,(\boldsymbol{T}_{(2i)}\cdot\boldsymbol{T}_{(ij)}) + X_3^0(1,i,2)\,(\boldsymbol{T}_{(1i)}\cdot\boldsymbol{T}_{(i2)})\Big]$$

Construct the real subtraction term,

$$\mathrm{d}\hat{\sigma}^S_{NLO} \sim \int_{\Phi_4} \langle\widetilde{\mathcal{M}}_3^0|\mathbb{X}_3^0|\widetilde{\mathcal{M}}_3^0\rangle$$

$$\mathrm{d}\hat{\sigma}^S_{NLO} \sim \int_{\Phi_4} \sum_{(i,j)\in(3,4)} \Big[X_3^0(1,i,j)\langle\widetilde{\mathcal{M}}_3^0|\widetilde{\mathcal{M}}_3^0\rangle + X_3^0(2,i,j)\langle\widetilde{\mathcal{M}}_3^0|\widetilde{\mathcal{M}}_3^0\rangle - \frac{1}{N^2}X_3^0(1,i,2)\langle\widetilde{\mathcal{M}}_3^0|\widetilde{\mathcal{M}}_3^0\rangle\Big]\, J_3^{(4)}$$

- same result as colour stripped approach

Colour explicit antenna subtraction @ NLO: summary

Strategy:

- construct virtual subtraction term from inserting $\boldsymbol{\mathcal{J}}^{(1)}(\epsilon)$
- construct real subtraction term from inserting $\mathbb{X}_3^0$
- evaluate explicitly in the most convenient colour basis

Features:

- can translate easily between the two approaches
- real subtraction constructed algorithmically without detailed knowledge of ME
- leading and sub-leading colour on same footing
- can be applied to arbitrary processes. . . automatable

Aside. . . spin correlations

Antennae are spin-averaged,

$$X_3^0(i,j,k)\,|\mathcal{M}_n^0|^2 \quad \xrightarrow{i||j\to g} \quad P^0_{ij\to g}(z)\,|\mathcal{M}_n^0|^2,$$

but $g \to gg$ and $g \to q\bar{q}$ splittings contain azimuthal correlations,

$$\begin{aligned} |\mathcal{M}_{n+1}^0|^2 \quad \xrightarrow{i||j\to g} & \quad P^{0,\mu\nu}_{ij\to g}(z)\,|\mathcal{M}_n^0|^2_{\mu\nu} \\ = & \quad P^0_{ij\to g}(z)\,|\mathcal{M}_n^0|^2 + \underbrace{\text{ang. terms}}_{\sim\cos(2\phi)} \end{aligned}$$

Pair up PS points related by $\Delta\phi = \pi/2$

- angular terms cancel
- antennae properly subtract matrix elements
- computational cost (also gain from improved convergence)
- difficult to match NLO method to parton showers etc

Tensorial antennae

We would like to have tensorial antennae for fully local subtraction,

$$\langle \cdots, \lambda_I, \cdots, \lambda_K \cdots | X^0_{3,I,I',K,K'}(i,j,k) | \cdots, \lambda_{I'}, \cdots, \lambda_{K'} \cdots \rangle$$

- define a generalized spin-correlation tensor $\Theta_{\mu\nu}$
- construct tensorial antennae, $X^0_{3,\mu\nu}(i,j,k)$, $X^0_{3,\mu\nu\rho\sigma}(i,j,k)$
  - tends to tensorial splitting function in collinear limit
  - yields spin averaged antenna when contracted with polarization tensor
  - no new integrals needed
- antennae promoted to matrices in spin space and colour space
- local subtraction for arbitrary processes @ NLO
- can a similar procedure be achieved @ NNLO?

Extending to NNLO - 2-loop singularities recap

$$|\mathcal{M}_n^2\rangle = \boldsymbol{I}^{(1)}(\epsilon)|\mathcal{M}_n^1\rangle + \boldsymbol{I}^{(2)}(\epsilon)|\mathcal{M}_n^0\rangle + \mathcal{O}(\epsilon^0)$$

$$\begin{aligned} \boldsymbol{I}^{(2)}(\epsilon) &= -\frac{1}{2}\boldsymbol{I}^{(1)}(\epsilon)\boldsymbol{I}^{(1)}(\epsilon) - \frac{\beta_0}{\epsilon}\boldsymbol{I}^{(1)}(\epsilon) \\ &+ \frac{e^{-\epsilon\gamma}\Gamma(1-2\epsilon)}{\Gamma(1-\epsilon)}\left(\frac{\beta_0}{\epsilon}+K\right)\boldsymbol{I}^{(1)}(2\epsilon) + \boldsymbol{H}^{(2)}(\epsilon) \end{aligned}$$

Leads to the two-loop ME pole structure,

$$\begin{aligned} |M_n^2|^2 &= \langle\mathcal{M}_n^1|2\mathrm{Re}\left[\boldsymbol{I}^{(1)}(\epsilon)\right]|\mathcal{M}_n^0\rangle + \langle\mathcal{M}_n^0|2\mathrm{Re}\left[\boldsymbol{I}^{(1)}(\epsilon)\right]|\mathcal{M}_n^1\rangle \\ &- \frac{1}{2}\langle\mathcal{M}_n^0|2\mathrm{Re}\left[\boldsymbol{I}^{(1)}(\epsilon)\right]^2|\mathcal{M}_n^0\rangle - \frac{\beta_0}{\epsilon}\langle\mathcal{M}_n^1|2\mathrm{Re}\left[\boldsymbol{I}^{(1)}(\epsilon)\right]|\mathcal{M}_n^0\rangle \\ &+ e^{-\epsilon\gamma}\frac{\Gamma(1-2\epsilon)}{\Gamma(1-\epsilon)}\left(\frac{\beta_0}{\epsilon}+K\right)\langle\mathcal{M}_n^0|2\mathrm{Re}\left[\boldsymbol{I}^{(1)}(2\epsilon)\right]|\mathcal{M}_n^0\rangle \\ &+ \langle\mathcal{M}_n^0|2\mathrm{Re}\left[\boldsymbol{H}^{(2)}(\epsilon)\right]|\mathcal{M}_n^0\rangle + \mathcal{O}(\epsilon^0) \end{aligned}$$

A dipole form for the Hard Function

Has the general form,

$$\boldsymbol{H}^{(2)}(\epsilon) = \sum_i C_i \boldsymbol{\mathcal{H}}_i^{(2)}(\epsilon) + \check{\boldsymbol{H}}^{(2)}(\epsilon) + \mathcal{O}(\epsilon)$$

- $\boldsymbol{\mathcal{H}}_i^{(2)}(\epsilon)$ diagonal in colour space
- $\check{\boldsymbol{H}}^{(2)}(\epsilon)$ a non-dipole tensor in colour space

} neither are colour dipoles

However...

$$\langle \mathcal{M}_n^0 | \check{\boldsymbol{H}}^{(2)}(\epsilon) | \mathcal{M}_n^0 \rangle = 0$$

then using colour conservation,

$$\boldsymbol{H}^{(2)}(\epsilon) = -\sum_{(i,j)} \boldsymbol{\mathcal{H}}_{ij}^{(2)}(\epsilon)\ (\boldsymbol{T}_i \cdot \boldsymbol{T}_j) + \text{irrelevant terms}$$

2-loop integrated antenna dipoles

Define a 2-loop insertion operator,

$$\boldsymbol{\mathcal{J}}^{(2)}(\epsilon) = \sum_{(i,j)} \boldsymbol{\mathcal{J}}_2^{(2)}(i,j)\ (\boldsymbol{T}_i \cdot \boldsymbol{T}_j)$$

$$\boldsymbol{\mathcal{J}}_2^{(2)}(i,j) \sim \mathcal{X}_4^0(s_{ij}) + \mathcal{X}_3^1(s_{ij}) + \frac{\beta_0}{\epsilon}\mathcal{X}_3^0\left[\left(\frac{s_{ij}}{\mu^2}\right)^{-\epsilon} - 1\right] + \mathcal{X}_3^0(s_{ij}) \otimes \mathcal{X}_3^0(s_{ij})$$

Double virtual subtraction term,

$$\begin{aligned} \mathrm{d}\hat{\sigma}^{U}_{NNLO} \quad &\sim \quad \langle \mathcal{M}_n^1 | \boldsymbol{\mathcal{J}}^{(1)}(\epsilon) | \mathcal{M}_n^0 \rangle + \langle \mathcal{M}_n^0 | \boldsymbol{\mathcal{J}}^{(1)}(\epsilon) | \mathcal{M}_n^1 \rangle \\ &+ \quad \langle \mathcal{M}_n^0 | \boldsymbol{\mathcal{J}}^{(1)}(\epsilon) \boldsymbol{\mathcal{J}}^{(1)}(\epsilon) | \mathcal{M}_n^0 \rangle \\ &+ \quad \langle \mathcal{M}_n^0 | \boldsymbol{\mathcal{J}}^{(2)}(\epsilon) | \mathcal{M}_n^0 \rangle \end{aligned}$$

Cascading down the calculation

Real-virtual involves the operators

$$\langle \mathcal{M}_{n+1}^{0} | \boldsymbol{\mathcal{J}}^{(1)}(\epsilon) | \mathcal{M}_{n+1}^{0} \rangle$$
$$\langle \widetilde{\mathcal{M}}_{n}^{0} | \mathbb{X}_{3}^{1} | \widetilde{\mathcal{M}}_{n}^{0} \rangle$$
$$\langle \widetilde{\mathcal{M}}_{n}^{1} | \mathbb{X}_{3}^{0} | \widetilde{\mathcal{M}}_{n}^{0} \rangle$$
$$\langle \widetilde{\mathcal{M}}_{n}^{0} | \boldsymbol{\mathcal{J}}^{(1)}(\epsilon) \otimes \mathbb{X}_{3}^{0} | \widetilde{\mathcal{M}}_{n}^{0} \rangle$$

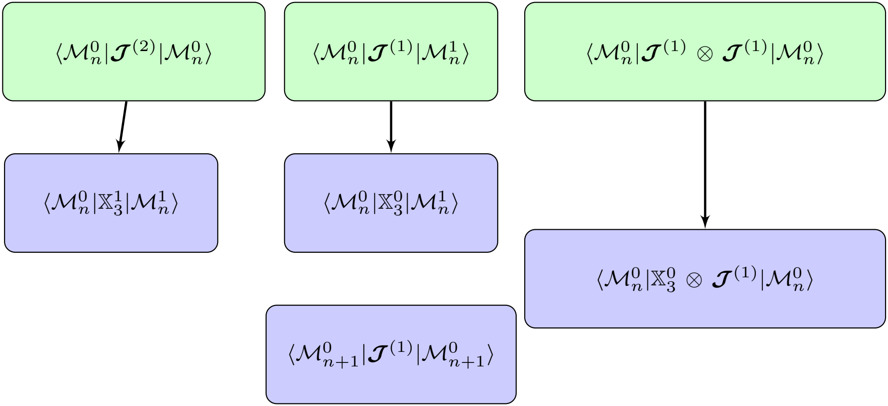
Double real built from,

$$\langle \widetilde{\mathcal{M}}_{n+1}^{0} | \mathbb{X}_{3}^{0} | \widetilde{\mathcal{M}}_{n+1}^{0} \rangle$$
$$\langle \widetilde{\mathcal{M}}_{n}^{0} | \mathbb{X}_{4}^{0} | \widetilde{\mathcal{M}}_{n}^{0} \rangle$$
$$\langle \widetilde{\mathcal{M}}_{n}^{0} | \mathbb{X}_{3}^{0} \otimes \mathbb{X}_{3}^{0} | \widetilde{\mathcal{M}}_{n}^{0} \rangle$$

$$\langle \mathcal{M}_n^0 | \boldsymbol{\mathcal{J}}^{(2)} | \mathcal{M}_n^0 \rangle$$

$$\langle \mathcal{M}_n^0 | \boldsymbol{\mathcal{J}}^{(1)} | \mathcal{M}_n^1 \rangle$$

$$\langle \mathcal{M}_n^0 | \boldsymbol{\mathcal{J}}^{(1)} \otimes \boldsymbol{\mathcal{J}}^{(1)} | \mathcal{M}_n^0 \rangle$$



$\langle \mathcal{M}_n^0 | \boldsymbol{\mathcal{J}}^{(2)} | \mathcal{M}_n^0 \rangle$
$\langle \mathcal{M}_n^0 | \boldsymbol{\mathcal{J}}^{(1)} | \mathcal{M}_n^1 \rangle$
$\langle \mathcal{M}_n^0 | \boldsymbol{\mathcal{J}}^{(1)} \otimes \boldsymbol{\mathcal{J}}^{(1)} | \mathcal{M}_n^0 \rangle$
$\langle \mathcal{M}_n^0 | \mathbb{X}_3^1 | \mathcal{M}_n^1 \rangle$
$\langle \mathcal{M}_n^0 | \mathbb{X}_3^0 | \mathcal{M}_n^1 \rangle$
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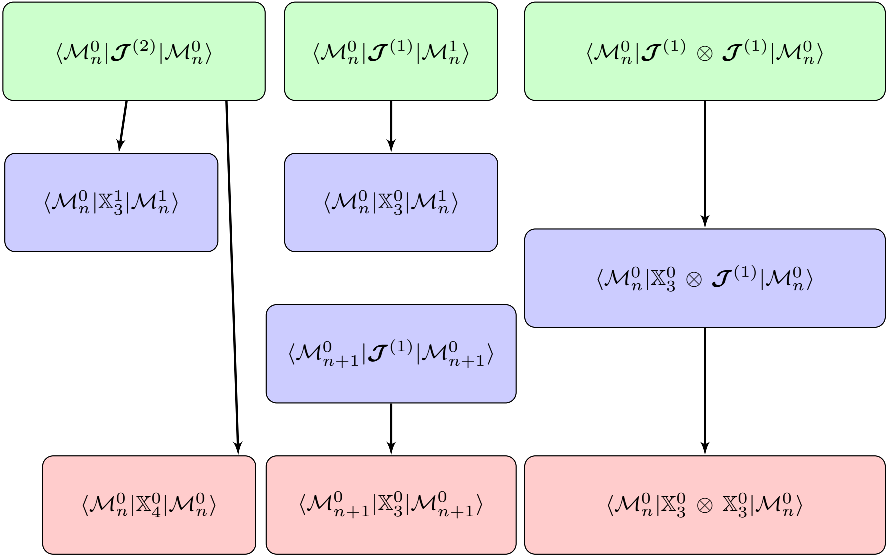
$\langle \mathcal{M}_{n+1}^0 | \boldsymbol{\mathcal{J}}^{(1)} | \mathcal{M}_{n+1}^0 \rangle$


$\langle \mathcal{M}_n^0 | \boldsymbol{\mathcal{J}}^{(2)} | \mathcal{M}_n^0 \rangle$
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$\langle \mathcal{M}_n^0 | \boldsymbol{\mathcal{J}}^{(1)} \otimes \boldsymbol{\mathcal{J}}^{(1)} | \mathcal{M}_n^0 \rangle$
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$\langle \mathcal{M}_{n+1}^0 | \boldsymbol{\mathcal{J}}^{(1)} | \mathcal{M}_{n+1}^0 \rangle$
$\langle \mathcal{M}_n^0 | \mathbb{X}_4^0 | \mathcal{M}_n^0 \rangle$
$\langle \mathcal{M}_{n+1}^0 | \mathbb{X}_3^0 | \mathcal{M}_{n+1}^0 \rangle$
$\langle \mathcal{M}_n^0 | \mathbb{X}_3^0 \otimes \mathbb{X}_3^0 | \mathcal{M}_n^0 \rangle$

Colourful subtraction - summary

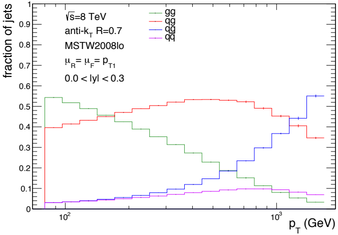
- Subtraction term can be written in colour space
- guided by universal singularity structure
- gives full matrix element subtraction, not just LC
- algorithmic and automatable

but. . . requires a full set of dipoles for a general scheme

- currently have all NLO dipoles
- all NNLO FF dipoles
- all NNLO FF, IF and II pure gluon dipoles
- many more, but not a complete set. . . yet!

NNLO calculations under way

- $pp \to jj$ [JC, Gehrmann De-Ridder, Gehrmann, Glover, Pires, Wells]
  - $gg \to jj$ leading colour ✓
  - $gg \to jj$ sub-leading colour ✓
  - $q\bar{q} \to jj$ leading colour ✓
  - $qg \to jj$ leading colour nearly there!
  - $gg \to jj$ leading N_F in preparation



LHC 8TeV

- $ep \to (2+1)j$ [JC, Gehrmann, Niehues]
- $pp \to H + j$ [Chen, Gehrmann, Glover, Jaquier]
- $pp \to V + j$ [JC, Gehrmann De-Ridder, Gehrmann, Glover, Morgan, Piebinga]

NNLO calculations under way

- $pp \to jj$ [JC, Gehrmann De-Ridder, Gehrmann, Glover, Pires, Wells]
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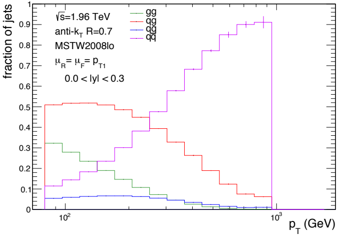


Tevatron

- $ep \to (2+1)j$ [JC, Gehrmann, Niehues]
- $pp \to H + j$ [Chen, Gehrmann, Glover, Jaquier]
- $pp \to V + j$ [JC, Gehrmann De-Ridder, Gehrmann, Glover, Morgan, Piebinga]

NNLO RR dijet subtraction terms

Constructed from three main contributions,

$$\mathrm{d}\hat{\sigma}^{S}_{NNLO} = \mathrm{d}\hat{\sigma}^{S,a}_{NNLO} + \mathrm{d}\hat{\sigma}^{S,b}_{NNLO} + \mathrm{d}\hat{\sigma}^{S,c,d,e}_{NNLO}$$

- a-term removes single unresolved divergence

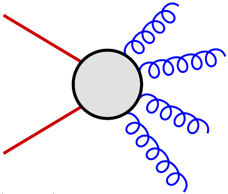
$$\mathrm{d}\hat{\sigma}^{S,a}_{NNLO} \sim X_3^0(i,j,k)\; M_5^0(\cdots, I, K, \cdots)$$

- b-term removes double unresolved divergence

$$\begin{aligned}\mathrm{d}\hat{\sigma}^{S,b}_{NNLO} &\sim X_4^0(i,j,k,l)\; M_4^0(\cdots, I, L, \cdots)\\ &- X_3^0(i,j,k)\; X_3^0(I,K,l)\; M_4^0(\cdots, I, L, \cdots)\\ &- X_3^0(j,k,l)\; X_3^0(i,J,L)\; M_4^0(\cdots, I, L, \cdots).\end{aligned}$$

- c,d,e-terms remove any spurious over-subtraction in a- and b-terms

Example, $q\bar{q} \to gggg$



Need to perform subtraction for

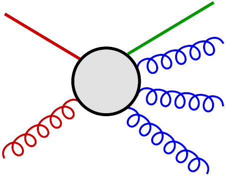
$$|M_6^0|^2 \sim \sum_{P(i,j,k,l)} M_6^0(1_q, i, j, k, l, 2_{\bar{q}})$$

Double unresolved limits subtracted using,

$$\begin{aligned} d\hat{\sigma}^b_{NNLO} \sim \sum \quad & + \quad D_4^0(1,i,j,k)\ M_4^0(\bar{1}, (\widetilde{ijk}), l, 2) \\ & + \quad F_4^0(i,j,k,l)\ M_4^0(1, (\widetilde{ijk}), (\widetilde{jkl}), 2) \\ & + \quad D_4^0(2,l,k,j)\ M_4^0(1, i, (\widetilde{jkl}), \bar{2}) \\ & - \quad \tilde{A}_4^0(1,i,k,2)\ M_4^0(\bar{1}, \tilde{j}, \tilde{l}, \bar{2}) \end{aligned}$$

- full subtraction term successfully removes all single and double unresolved divergence

Quark-gluon channel: identity changing collinear limits



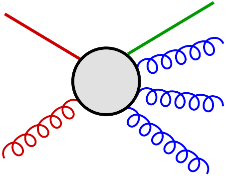
Need to perform subtraction for

$$|M_6^0|^2 \sim \sum_{P(2,i,j,k)} M_6^0(1_q, 2_g, i, j, k, Q)$$

Matrix element can collapse onto different initial states

- quark-gluon, e.g., $2|i|j$, $i|j|k$, $Q|i|j$ etc
- quark-antiquark e.g., $2|i|Q$ etc
- gluon-gluon e.g. $1|i|Q$ etc

Quark-gluon channel: identity changing collinear limits



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Matrix element can collapse onto different initial states

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- quark-antiquark e.g., $2|i|Q$ etc
- gluon-gluon e.g. $1|i|Q$ etc

But subtraction term must make a choice

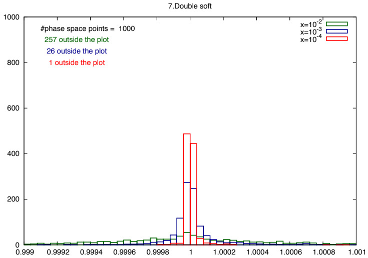
$$D_4^0(Q, i, j, 2)\ M_4^0(1, k, \bar{2}, (\widetilde{ijQ}))$$

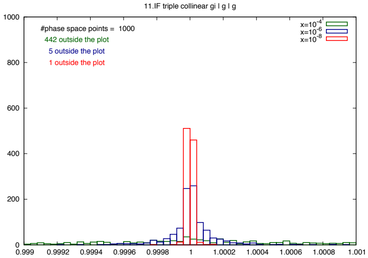
or

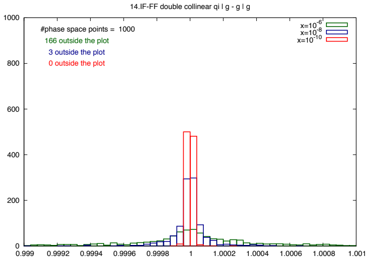
$$D_4^0(Q, i, j, 2)\ M_4^0(1, k, (\widetilde{ijQ}), \bar{2})$$

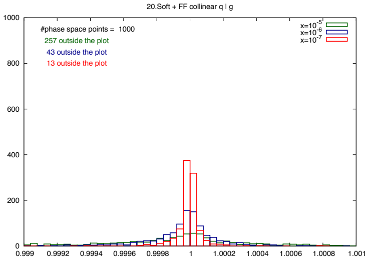
- many spurious divergences

Double real quark-gluon channel tests









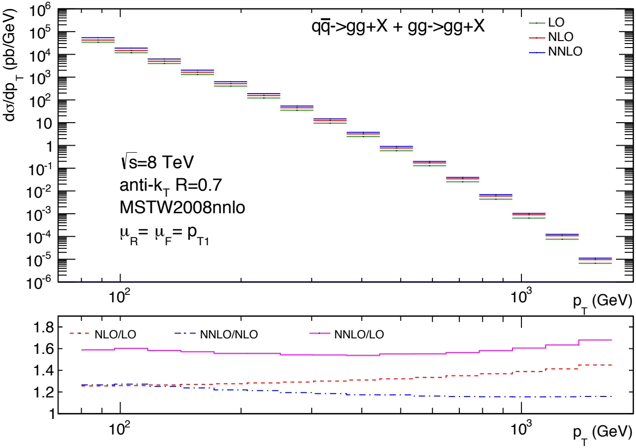
Preliminary dijet results

Preliminary results for full-colour “gluons only” scattering and leading colour $q\bar{q}$ scattering combined

Numerical setup and cuts:

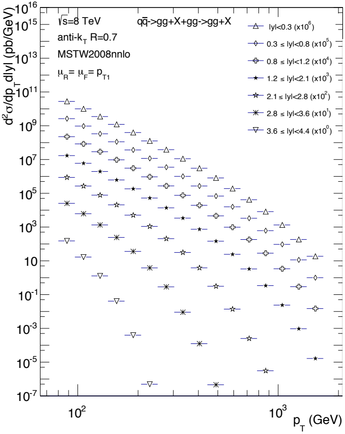
- leading jet transverse momentum $p_{T_1} > 80$ GeV
- all other jets with at least $p_T > 60$ GeV
- jets with rapidities $|y| < 4.4$ considered
- anti-k_T jet algorithm with $R = 0.7$
- all scales taken to be common dynamical scale $\mu = p_{T_1}$
- MSTW2008NNLO PDF set

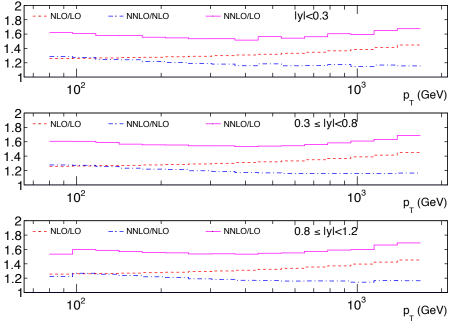
Inclusive jet p_T distribution



- NNLO correction between $\sim 15\%$ and 26% w.r.t NLO
- K-factor at high p_T brought under control

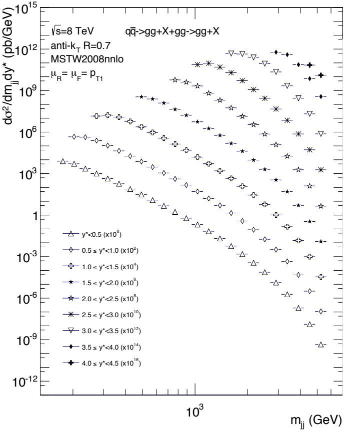
Double differential inclusive jet p_T distribution

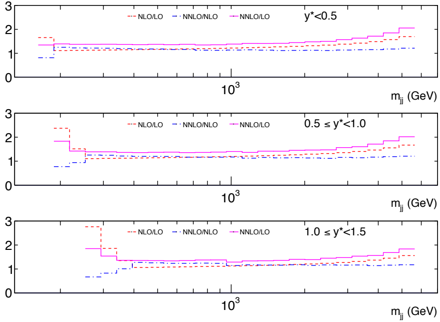




- NNLO correction between $\sim 15\%$ and 26% w.r.t NLO
- similar effects in other rapidity slices

Double differential exclusive dijet distribution

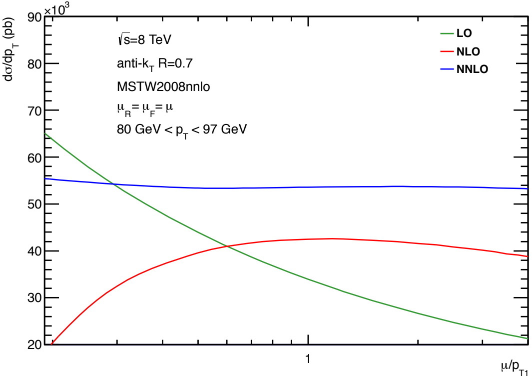




- NNLO correction $\sim 20\%$ w.r.t NLO
- similar effects in other y^* slices

Inclusive jet p_T scale dependence

Full colour gluons only contribution



Summary

Antenna subtraction is one of the most powerful and versatile methods for NNLO computations

- allows hadronic initial states
- can cope with several final-state jets
- analytic pole cancellation
- colour explicit formalism generalizes to arbitrary processes and SLC
- many calculations under way
  - expect some preliminary dijet results soon
- new generalizations of the method in progress

Thank you for your attention!