Scattering Amplitudes and Top Phenomenology with OpenLoops

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based on

F. Cascioli, P. Maierhöfer and S.P., PRL **108** (2012) 111601 [arXiv:1111.5206]

F. Cascioli, S. Höche, F. Krauss, P. Maierhöfer, S. P. and F. Siegert, arXiv:1309.0500

F. Cascioli, P. Maierhöfer, N. Moretti, S. P. and F. Siegert, arXiv:1309.5912

F. Cascioli, S. Kallweit, P. Maierhöfer and S. P., arXiv:1312.0546

Humboldt University, Berlin, 30 January 2014

Outline of the talk

(A) Introduction

(B) OpenLoops algorithm

(C) A unified NLO description of $\mathrm{t\bar{t}}$ and Wt production

(D) MC@NLO matching for $\mathrm{t\bar{t}b\bar{b}}$ production with $m_{\mathrm{b}} > 0$

(A) Introduction

NLO Revolution and Automation

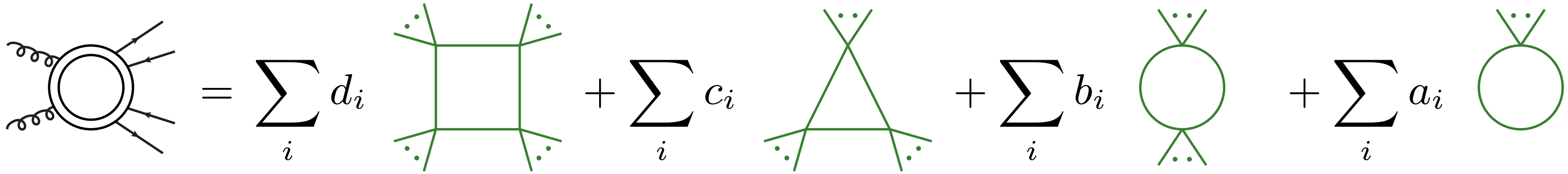
NLO QCD calculations for $2 \to 4(5,6)$ processes at the LHC

- many recent results (2009-2013): $5j$, $W + 5j$, $Z + 4j$, $H + 3j$, $WWjj$, $WZjj$, $\gamma\gamma + 3j$, $W\gamma\gamma j$, $WWb\bar{b}$, $b\bar{b}b\bar{b}$, $t\bar{t}b\bar{b}$, $t\bar{t}jj$, $t\bar{t}t\bar{t}$, ...
- NLO wish list closed since 2→4 NLO *feasibility* well established (... but various results still incomplete ...)
- serious multi-particle simulations important for Run 2 ⇒ emphasis should move from proof-of-concept papers to complete simulations and nontrivial pheno studies
- technical frontier just shifted and still exciting to explore

NLO automation including matching and merging

- many tools: `CutTools`, `Samurai`, `HELAC-NLO`, `MadLoop`, `GoSam`, `BlackHat`, `NGluon`, `OpenLoops`, `Collier`, `Recola`, `MADGRAPH/aMC@NLO`, `POWHEG`, `Sherpa`, `Herwig`, `Pythia`
- new attitude towards R&D at NLO: think more in terms of general methodological features (e.g. EW corrections) and less in terms of single processes
- ... keeping in mind that simulation of every single process needs to be well understood and some processes will require more than "vanilla NLO"
- methodology and phenomenology at NLO much more involved wrt LO: usage, maintanance and development of tools requires much higher level of expertise and TH/EXP cross-talk
- algorithmic efficiency crucial in order to promote NLO to the default accuracy in LHC studies ⇒ don't stop R&D

(B) The OpenLoops Algorithm [Cascioli, Maierhöfer, S.P '11]



OpenLoops Generator [Cascioli, Maierhöfer, S.P., PRL **108** (2012) 111601]

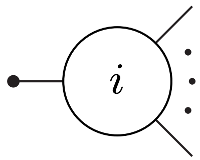
- fully automated generation of tree and loop amplitudes for NLO (with UV/IR CTs)
- conceived to break **multi-particle** bottlenecks (fast, stable, flexible)
- NLO QCD for $2 \to 2, 3, 4$ SM processes ($2 \to 5$ and NLO EW possible)

Hybrid "tree–loop" algorithmic approach

- constructs process-dependent 1-loop ingredients with hybrid "tree–loop" approach based on diagrammatic building blocks (openloops)
- pinch relations to obtain n-point diagrams from $(n-1)$-point diagrams
- works in combination with both tensor-integral and OPP reduction
- numerical recursion inspired by 1-loop Dyson-Schwinger recursion [van Hameren '09]

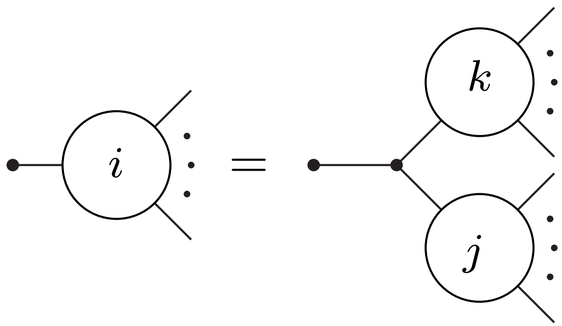
Tree generator

Colour-stripped tree diagrams are built numerically in terms of sub-trees

$w^{\beta}(i) =$ 

$\beta \leftrightarrow$ off-shell line spin

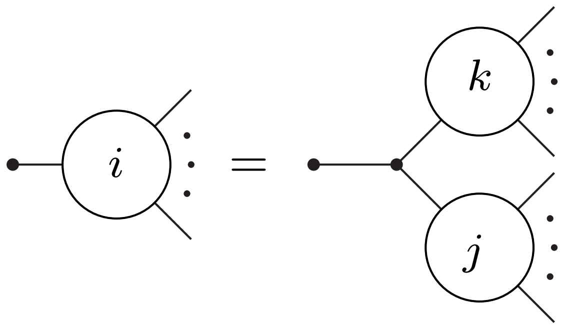
and recursively merged by attaching vertices and propagators



$$w^{\beta}(i) = \frac{X^{\beta}_{\gamma\delta}(i,j,k)}{p_i^2 - m_i^2}\, w^{\gamma}(j)\, w^{\delta}(k)$$

Completely generic and automatic (similar to `Madgraph`+`HELAS`)

- **flexible** (only $\mathcal{L}_{\text{int}}$ dependent)
- **fast** (many diagrams share *common sub-trees*)
- **efficient colour bookkeeping** (colour factorisation and algebraic reduction)



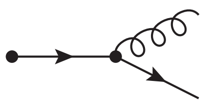
$$w^{\beta}(i) = \frac{X^{\beta}_{\gamma\delta}(i,j,k)}{p_i^2 - m_i^2}\, w^{\gamma}(j)\, w^{\delta}(k)$$

sub-tree $=$ individual topology with off-shell line $\neq$ off-shell current

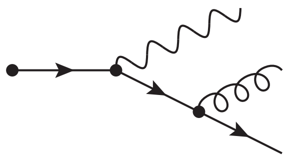
Example

$w_{\alpha}(1) =$  $= \bar{u}_{\alpha}(p_1, \lambda_1)$

$w_{\mu}(2) =$  $= \epsilon^{*}_{\mu}(p_2, \lambda_2)$

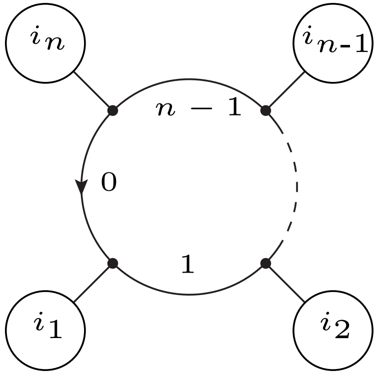
$w_{\beta}(12) =$  $= \dfrac{g_{\mathrm{S}}\left[(\not{p}_{12} + m)\gamma^{\mu}\right]_{\alpha\beta}}{p_{12}^2 - m^2}\, w_{\alpha}(1)\, w_{\mu}(2)$

$w_{\nu}(3) =$  $= \epsilon^{*}_{\nu}(p_3, \lambda_3)$

$w_{\gamma}(123) =$  $= \dfrac{e\left[(\not{p}_{123} + m)\gamma^{\nu}(1-\gamma_5)\right]_{\beta\gamma}}{2\sqrt{2}s_{\mathrm{W}}(p_{123}^2 - m^2)}\, w_{\beta}(12)\, w_{\nu}(3)$ etc.

Recursion terminates when full set of diagram can be obtained via sub-diagram merging

Colour-stripped loop diagrams (and reduction to basis integrals)



$$= \int \frac{\mathrm{d}^D q\, \mathcal{N}(\mathcal{I}_n; q)}{D_0 D_1 \dots D_{n-1}} = \sum_{r=0}^{R} \mathcal{N}_{\mu_1 \dots \mu_r}(\mathcal{I}_n) \underbrace{\int \frac{\mathrm{d}^D q\, q^{\mu_1} \dots q^{\mu_r}}{D_0 D_1 \dots D_{n-1}}}_{\text{tensor integral}}$$

`OpenLoops` computes *symmetrised* $\mathcal{N}_{\mu_1 \dots \mu_r}(\mathcal{I}_n)$ *coefficients*

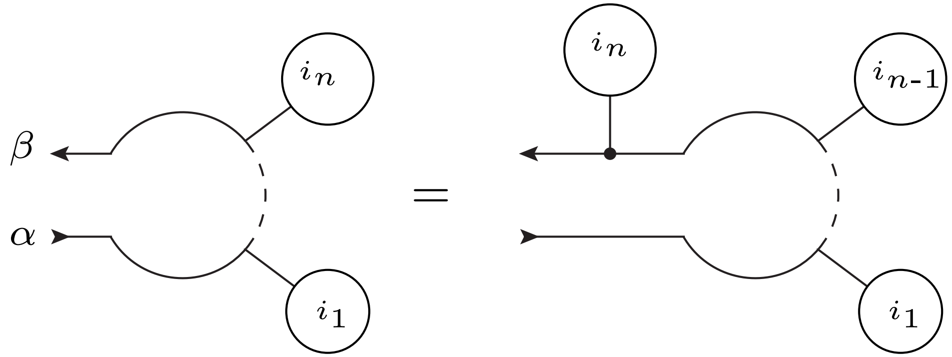
tensor-rank	R	0	1	2	3	4	5	6
# coeff. per diagram	$\binom{R+4}{4}$	1	5	15	35	70	126	210

(ranks 4–6: 6 particles)

and applies **two alternative reductions:**

(A) **Tensor-integral reduction** [Denner/Dittmaier '05] **avoids instabilities** (Gram-determinant expansions)

(B) **OPP reduction** [Ossola, Papadopolous, Pittau '07] based on numerical evaluation of $\mathcal{N}(\mathcal{I}_n; q) = \sum \mathcal{N}_{\mu_1 \dots \mu_r}(\mathcal{I}_n)\, q^{\mu_1} \dots q^{\mu_r}$ at multiple q-values (**strong speed-up!**)



Tree generators for "usual" OPP-input $\mathcal{N}(\mathcal{I}_n; q)$

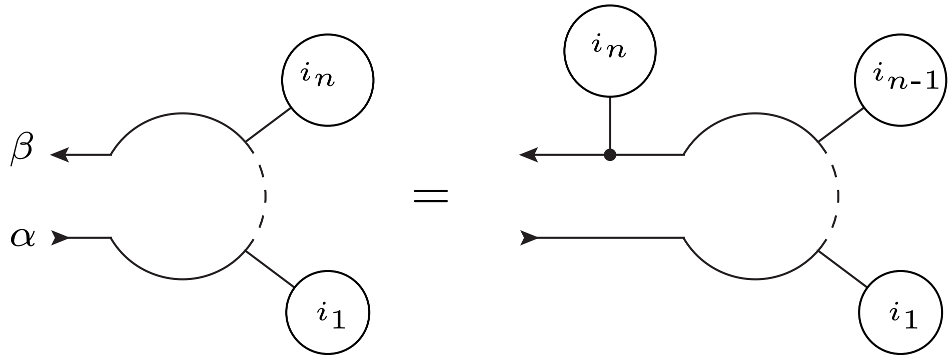
Cut-open loops can be built by recursively attaching external sub-trees

$$\mathcal{N}^{\beta}_{\alpha}(\mathcal{I}_n; q) = X^{\beta}_{\gamma\delta}(\mathcal{I}_n, i_n, \mathcal{I}_{n-1})\,\mathcal{N}^{\gamma}_{\alpha}(\mathcal{I}_{n-1}; q)\, w^{\delta}(i_n)$$

like in **conventional tree generators**

- one-loop automation in `Helac-NLO` (off-shell recursion) and `MadLoop` (diagrams)
- CPU expensive OPP reduction (multiple-q evaluations) since *tree algorithms conceived for fixed momenta*

Nature of loop amplitudes requires loop-momentum *functional* dependence!



OpenLoops recursion for $\mathcal{N}^{\beta}_{\mu_1\ldots\mu_r;\alpha}(\mathcal{I}_n)$

Handle building blocks of recursion as *polynomials* in the loop momentum q

$$\underbrace{\mathcal{N}^{\beta}_{\alpha}(\mathcal{I}_n;q)}_{\sum_{r=0}^{n}\mathcal{N}^{\beta}_{\mu_1\ldots\mu_r;\alpha}(\mathcal{I}_n)\,q^{\mu_1}\ldots q^{\mu_r}} \quad = \quad \underbrace{X^{\beta}_{\gamma\delta}(\mathcal{I}_n,i_n,\mathcal{I}_{n-1})}_{Y^{\beta}_{\gamma\delta}+q^{\nu}\,Z^{\beta}_{\nu;\gamma\delta}}\;\underbrace{\mathcal{N}^{\gamma}_{\alpha}(\mathcal{I}_{n-1};q)}_{\sum_{r=0}^{n-1}\mathcal{N}^{\beta}_{\mu_1\ldots\mu_r;\alpha}(\mathcal{I}_{n-1})\,q^{\mu_1}\ldots q^{\mu_r}}\;w^{\delta}(i_n)$$

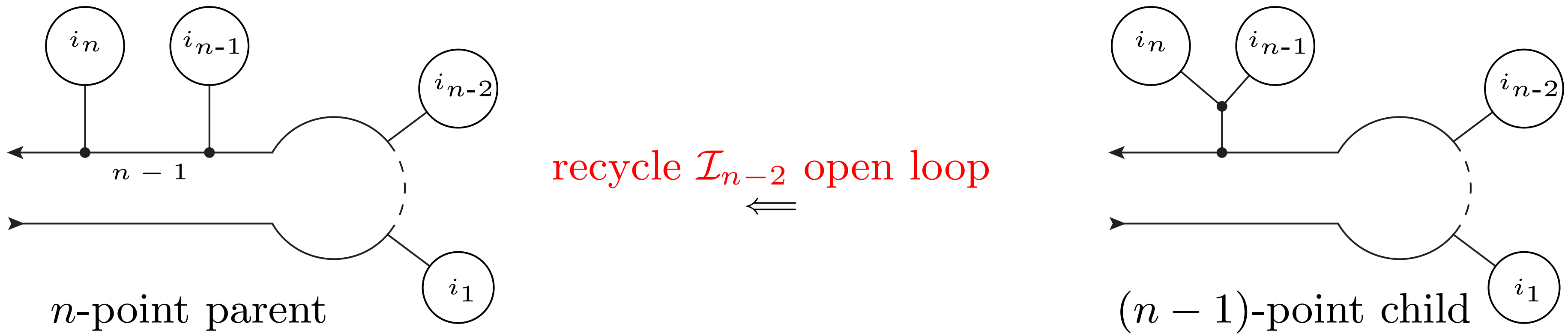
and construct polynomial coefficients with “open loops recursion”

$$\mathcal{N}^{\beta}_{\mu_1\ldots\mu_r;\alpha}(\mathcal{I}_n)=\left[Y^{\beta}_{\gamma\delta}\,\mathcal{N}^{\gamma}_{\mu_1\ldots\mu_r;\alpha}(\mathcal{I}_{n-1})+Z^{\beta}_{\mu_1;\gamma\delta}\,\mathcal{N}^{\gamma}_{\mu_2\ldots\mu_r;\alpha}(\mathcal{I}_{n-1})\right]\;w^{\delta}(i_n)$$

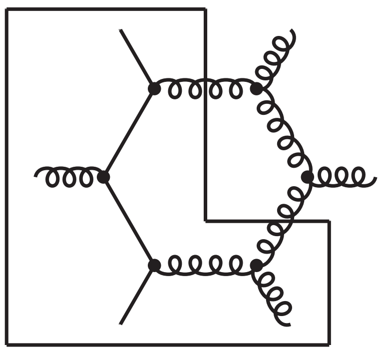
Key features

- **tree-like recursion** supplemented with **complete loop-momentum information**

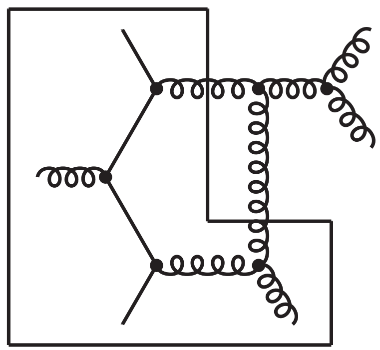
- **fully flexible and automated** (universal kernels dictated by Feynman rules)
- **very fast thanks to**:
  - optimal implementation
  - helicity/colour/loop decoupling
  - pinch relations: n-point loop diagrams can be obtained starting from pre-computed $(n-1)$-point child diagrams



Example



6-point parent

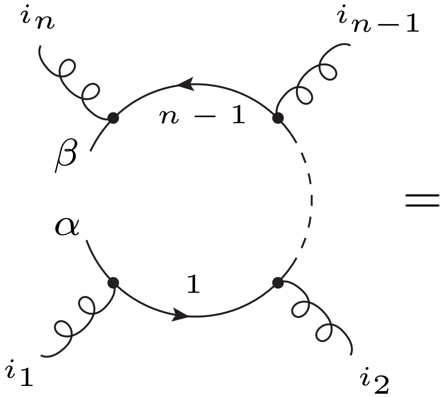


5-point child

Complicated diagrams require only “last missing piece” (always works in QCD!)

Example of OpenLoops recursion for a fermionic loop

$$\mathcal{N}^{\beta}_{\alpha}(\mathcal{I}_n; q) =$$



$$= g_S\left[(\not{q} + \not{p}_n + m)\gamma^{\nu}\right]_{\beta\gamma}\, \mathcal{N}^{\gamma}_{\alpha}(\mathcal{I}_{n-1}; q)\, \varepsilon^*_{\nu}(p_n, \lambda_n)$$

- n-point open-loop coefficients of rank $r = 0, 1, \ldots, n$

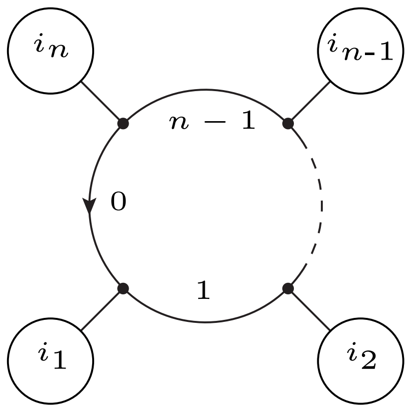
$$\begin{aligned}
\mathcal{N}^{\beta}_{;\alpha}(\mathcal{I}_n) &= g_S\left[(\not{p}_n + m)\gamma^{\nu}\right]_{\beta\gamma}\, \mathcal{N}^{\gamma}_{;\alpha}(\mathcal{I}_{n-1})\, \varepsilon^*_{\nu}(p_n, \lambda_n) \\
\mathcal{N}^{\beta}_{\mu_1;\alpha}(\mathcal{I}_n) &= g_S\left\{\left[(\not{p}_n + m)\gamma^{\nu}\right]_{\beta\gamma}\, \mathcal{N}^{\gamma}_{\mu_1;\alpha}(\mathcal{I}_{n-1}) + \left[\gamma_{\mu_1}\gamma^{\nu}\right]_{\beta\gamma}\, \mathcal{N}^{\gamma}_{;\alpha}(\mathcal{I}_{n-1})\right]\, \varepsilon^*_{\nu}(p_n, \lambda_n) \\
&\text{etc.}
\end{aligned}$$

- initial condition for 0-point rank-0 open loop

$$\mathcal{N}^{\gamma}_{;\alpha}(\mathcal{I}_0) = \delta^{\gamma}_{\alpha}$$

- rank, i.e. complexity, increases with n $\Rightarrow$ symmetrised $\mu_1 \ldots \mu_r$ components!
- bookkeeping of tensor components fully automated

R_2 Rational Terms

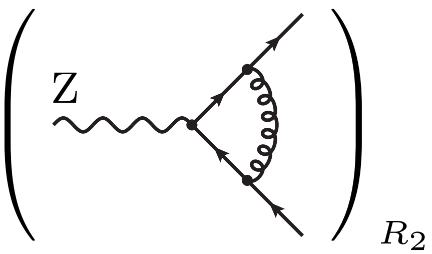


$$= \sum_{r=0}^{R} \underbrace{\mathcal{N}_{\mu_1\ldots\mu_r}(\mathcal{I}_n)}_{\text{in } D=4} \int \frac{\mathrm{d}^D q\; q^{\mu_1}\ldots q^{\mu_r}}{D_0 D_1 \ldots D_{n-1}}$$

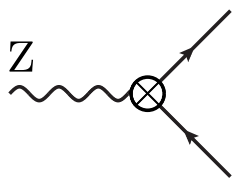
Extra rational terms from $3 < \mu_1, \ldots, \mu_r \leq D-1$ coefficient components

$$R_2 = \sum_{\mu_1\ldots\mu_r=0}^{D-1} \mathcal{N}_{\mu_1\ldots\mu_r}\Big|_{D=4-2\varepsilon} T_{\mathrm{UV}}^{\mu_1\ldots\mu_r} - \sum_{\mu_1\ldots\mu_r=0}^{3} \mathcal{N}_{\mu_1\ldots\mu_r}\Big|_{D=4} T_{\mathrm{UV}}^{\mu_1\ldots\mu_r}$$

From catalogue of 2-, 3- and 4-point 1PI diagrams (depends only on model)



$=$



$$= -\frac{g_{\mathrm{S}}^2}{16\pi^2}\frac{N_c^2-1}{2N_c}\gamma^\mu(g_{\mathrm{V}}^{\mathrm{Z}} - g_{\mathrm{A}}^{\mathrm{Z}}\gamma_5)$$

etc.

Draggiotis, Garzelli, Malamos, Papadopoulos, Pittau ‘09–‘11 Shao, Zhang, Chao ‘11

OpenLoops Implementation and Technical Features

One-loop QCD corrections to SM processes fully automated

- process-definition file $\Rightarrow$ `Fortran` 90 libraries for matrix elements

Other technical features

- interfaced to `Collier` library [Denner, Dittmaier, Hofer] for tensor integrals
- on-the-fly quadruple precision (very useful for benchmarks and NNLO)
- loop-induced processes
- speed of tree amplitudes optimised
- precision checks against independent in-house generator for > 100 processes
- ...

Flexibility and Automation

Process	size [MB]	t_{code} [s]
$u\bar{u} \to t\bar{t}$	0.1	2.2
$u\bar{u} \to W^+W^-$	0.1	7.2
$u\bar{d} \to W^+g$	0.1	4.2
$gg \to t\bar{t}$	0.2	5.4
$u\bar{u} \to t\bar{t}g$	0.4	12.8
$u\bar{u} \to W^+W^-g$	0.4	39.8
$u\bar{d} \to W^+gg$	0.5	22.9
$gg \to t\bar{t}g$	1.2	52.9
$u\bar{u} \to t\bar{t}gg$	3.6 (200)*	236 ($\sim 10^6$)*
$u\bar{u} \to W^+W^-gg$	2.5 (1000)*	381.7 ($\sim 10^6$)*
$u\bar{d} \to W^+ggg$	4.2	366.2
$gg \to t\bar{t}gg$	16.0	3005

Compact code

- 100 kB to few MB object files
- $\mathcal{O}(10^2$–$10^3)$ compression in $2 \to 4$

Fast code generation/compilation

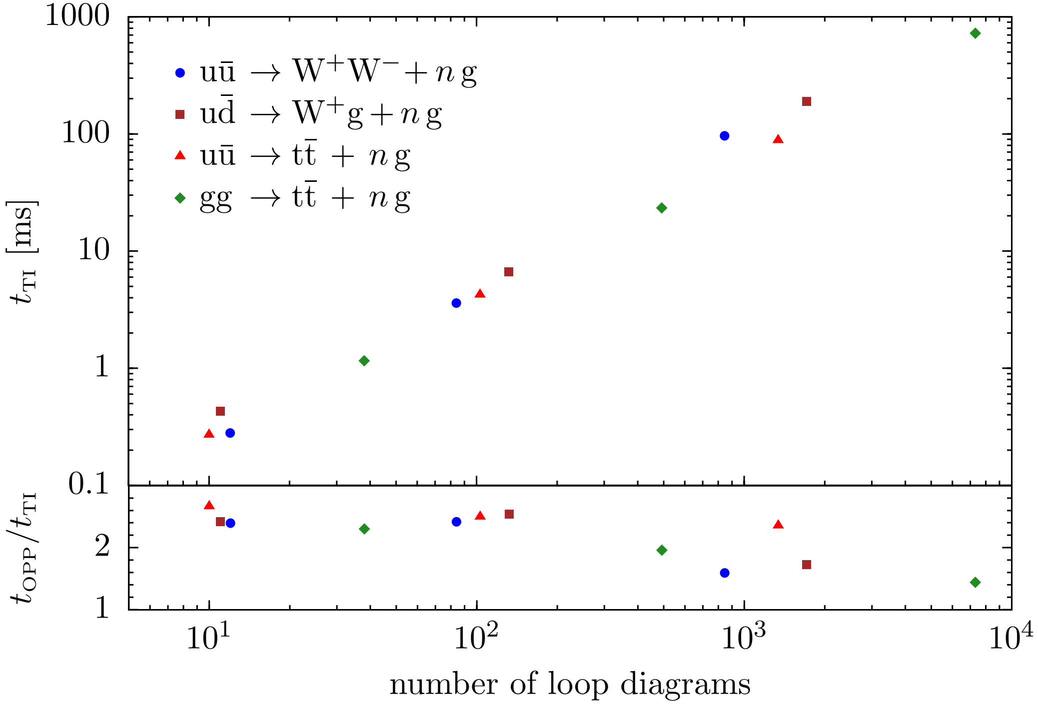
- few seconds to minutes
- $\mathcal{O}(10^3)$ speed-up in $2 \to 4$

Large-scale applicability!

*pp $\to$ $\mathrm{t\bar{t}b\bar{b}}$ & $\mathrm{WWb\bar{b}}$ (Bredenstein, Denner, Dittmaier, Kallweit and S.P. ‘09–‘11)

High CPU efficiency for multi-particle processes

Timings including col/hel sums (`Intel i5-750` core)



$2 \to 4$ amplitudes

- $\mathcal{O}(10^3)$ diagrams in $\mathcal{O}(10^2)$ ms/point
- competitive with fastest codes

Scaling

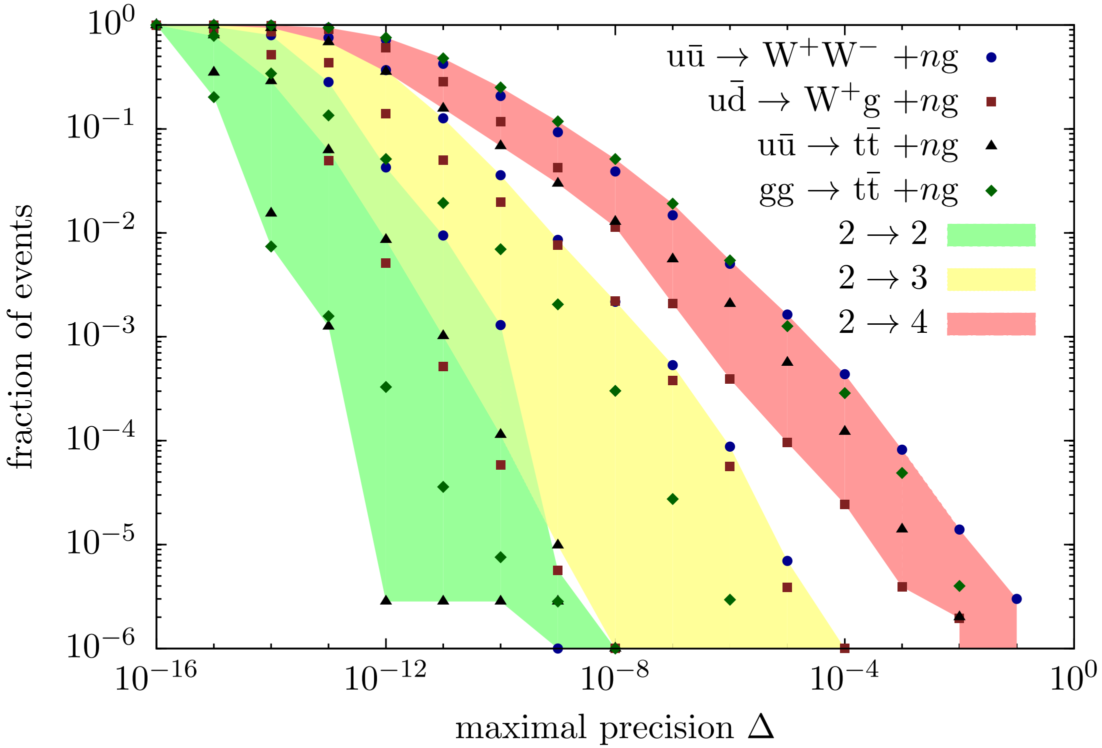
- linear $n_{\rm diag}$-scaling $\Rightarrow$ $\mathcal{O}(10^5)$ diagrams feasible
- factor 20 per extra leg $\Rightarrow$ $2 \to 5$ feasible

Tensor-reduction vs OPP

- similar timings with `OpenLoops`!

Numerical stability with tensor reduction in double precision

Stability Δ in samples of 10^6 points ($\sqrt{\hat{s}} = 1\,\mathrm{TeV}$, $p_\mathrm{T} > 50\,\mathrm{GeV}$, $\Delta R_{ij} > 0.5$)



Average number of correct digits

- 11-15

Cross section accuracy

- depends on tails
- stability issues grow with n_part

$2 \to 4$ processes very stable

- $\lesssim 0.01\%$ prob. that $\Delta_\mathrm{S} < 10^{-3}$
- thanks to Gram-determinant expansions in `Collier`!

Real-life NLO applications

- $\mathcal{O}(10^{-4})$ unstable points in most challenging $2 \to 4$ calculations considered so far
- can be monitored and safely suppressed thanks to **online instability-trigger**

Interfacing OpenLoops with NLO Monte-Carlo Tools

Interface with various MC tools (IR subtraction, integration) provide complete automation from process definition to hadron-collider observables

- **dedicated interface to Sherpa2.0**
  - automated matching (MC@NLO) to `Sherpa` shower and multi-jet merging (MEPS@NLO)
- **parton-level Monte-Carlo by S. Kallweit**
  - fully automated and very fast MC integrator
- **standard BLHA interface**
  - applicable to any other Monte-Carlo tool
  - completed very recently in combination with Herwig++ and now under validation

First OpenLoops Applications

Recent papers

- MEPS@NLO for $\ell\ell\nu\nu$+0,1 jets, Cascioli, Höche, Krauss, Maierhöfer, S. P. and Siegert, arXiv:1309.0500
- MC@NLO for $\mathrm{pp} \to \mathrm{t\bar{t}b\bar{b}}$ with $m_\mathrm{b} > 0$, Cascioli, Maierhöfer, Moretti, S. P. and Siegert, arXiv:1309.5912
- NLO for $\mathrm{pp} \to \mathrm{W^+W^-b\bar{b}}$ with $m_\mathrm{b} > 0$, Cascioli, Kallweit, Maierhöfer and S. P., arXiv:1312.0546
- NNLO for $\mathrm{pp} \to \gamma Z$ production, Grazzini, Kallweit, Rathlev and Torre, arXiv:1309.7000
- NLO merging for $\mathrm{pp} \to \mathrm{HH}$+0,1 jets, Maierhöfer and Papaefstathiou, arXiv:1401.0007

General motivation

- Higgs phenomenology
- technical stress tests for OpenLoops: multi-particle and multi-scale processes, loop-induced processes, multiple resonances, . . .
- beyond parton-level NLO: MC@NLO, MEPS@NLO and NNLO applications

Publication Plans and Process Library

Towards OpenLoops publication

- all technical prerequisites essentially fulfilled: many processes validated, good experience in challenging real-life applications, BLHA interface almost ready
- we aim at code release in early 2014

The release is planned as NLO QCD library for $2 \to 2, 3, 4$ processes

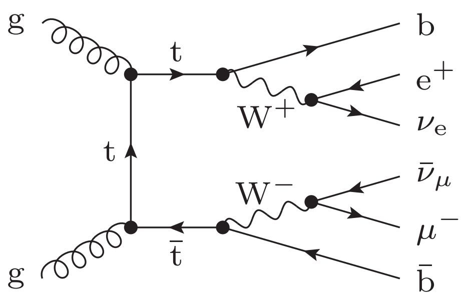
- first version already available to MCWGs of ATLAS/CMS
- new processes can/will be easily added (also upon user request)

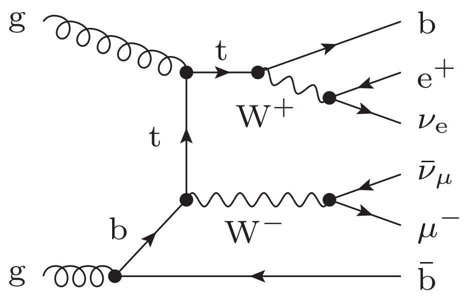
W/Z	γ	jets	HQ pairs	single-top	Higgs
V+3j	γ+3j	3(4)j	$\mathrm{t\bar{t}}$+2j	tb+1j	(H+2j)
VV+2j	$\gamma\gamma$+2j		$\mathrm{t\bar{t}b\bar{b}}$	t+1(2)j	VH+1j
gg $\to VV$+1j	$V\gamma$+2j		$\mathrm{t\bar{t}}V$+1j	tW+0(1)j	$\mathrm{t\bar{t}H}+1j$
VVV+1j			$\mathrm{b\bar{b}}V$+1j		$qq \to \mathrm{H}qq$+0(1)j
gg $\to VVV$					

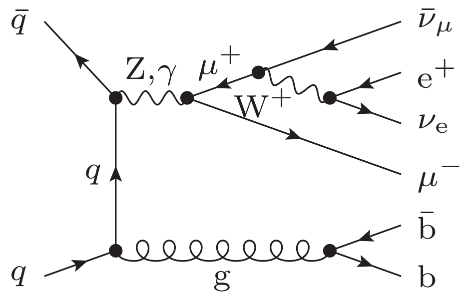
lower jet multiplicities implicitly understood

(C) Unified $\mathrm{t\bar{t}}$ and Wt description at NLO [Cascioli,Kallweit,Maieröfer,S.P. '13]

Top-pair production plus (di-leptonic) decay at NLO







NWA [Bernreuther et al. '04; Melnikov, Schulze '09]

- Only $t\bar{t}$ channels in $\Gamma_t \to 0$ limit

$pp \to W^+W^-b\bar{b}$ **in 5F scheme** [Denner, Dittmaier, Kallweit, S.P. '10; Bevilacqua et al. '10; Heinrich et al. '13]

- off-shell, single- and non-resonant contributions
- small $\mathcal{O}(\Gamma_t/m_t)$ effects for "inclusive" $t\bar{t}$ cuts
- $m_b = 0$ approx. requires two hard b-jets ($g \to b\bar{b}$ collinear singularities)

$pp \to W^+W^-b\bar{b}$ **in 4F scheme** ($m_b > 0$)

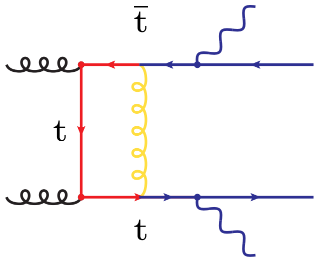
[Frederix'13; Cascioli,Kallweit,Maieröfer,S.P. '13]

- full b-quark phase space
- first consistent $t\bar{t}$ and Wt combination with interference at LO and NLO $\Rightarrow$ **Wt contribution pert. stable**
- important for top-backgrounds in 0- and 1-jet bins (e.g. in $H \to WW$)
- challenging multi-particle, multi-resonance, multi-scale $(m_b, \ldots, m_{t\bar{t}})$ process

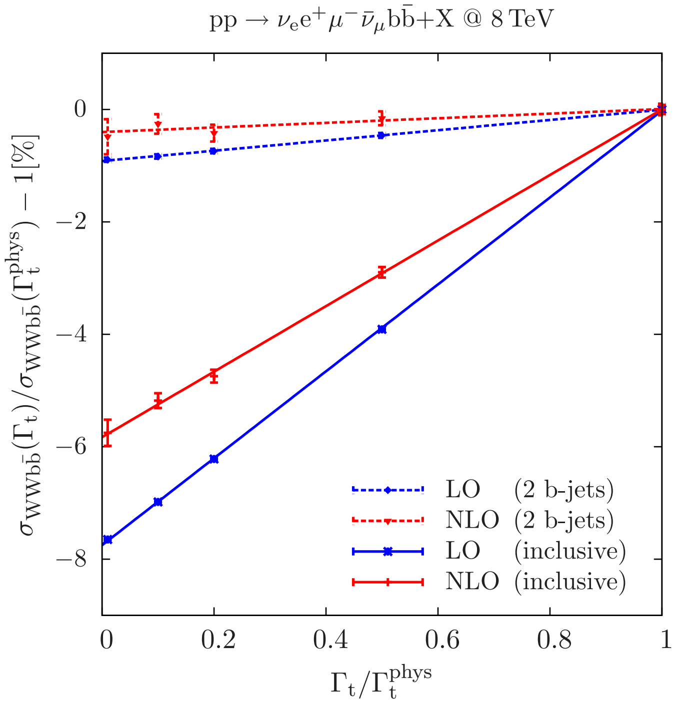
ill-defined $\mathrm{t\bar{t}}/\mathrm{Wt}$ separation in 5F scheme $\Rightarrow$ gauge-invariant $\mathrm{t\bar{t}}$/non-$\mathrm{t\bar{t}}$ separation

Numerical NWA $\Rightarrow$ on-shell $\mathrm{t\bar{t}}$ production$\times$decay

$$\mathrm{d}\sigma_{\mathrm{t\bar{t}}} = \lim_{\Gamma_{\mathrm{t}}\to 0} \left(\frac{\Gamma_{\mathrm{t}}}{\Gamma_{\mathrm{t}}^{\mathrm{phys}}}\right)^2 \mathrm{d}\sigma_{\mathrm{W^+W^-b\bar{b}}}(\Gamma_{\mathrm{t}})$$



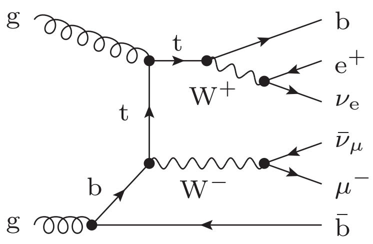
permille-level convergence shows cancellation of soft-gluon $\ln(\Gamma_{\mathrm{t}}/m_{\mathrm{t}})$ singularities



Finite-top-width remainder (FtW)

- contains all $\mathcal{O}(\Gamma_{\mathrm{t}}/m_{\mathrm{t}})$ effects: off-shell $\mathrm{t\bar{t}}$ production, single-top and non-resonant contributions with interferences
- from sub-percent for 2 b-jet final states to 6–8% effect in inclusive case (and more for 0/1-jets!)

Ad-hoc dynamic scale choice for multi-channel/multi-scale nature of $\mathrm{W^+W^-b\bar{b}}$



Idea: $\mu_\mathrm{R} \sim m_\mathrm{t}$ for $\mathrm{g} \to \mathrm{b\bar{b}}$ splittings might generate corrections up to $\alpha_S(m_\mathrm{b})/\alpha_S(m_\mathrm{t}) \sim 2$ in Wt contribution

Appropriate scales for $\mathrm{t\bar{t}}$ and Wt production (see CKKW and AP evolution)

$$\mu^2_{\mathrm{t\bar{t}}} = E_{\mathrm{T,t}} E_{\mathrm{T,\bar{t}}} \qquad \mu^2_{\mathrm{tW^-}} = E_{\mathrm{T,t}} E_{\mathrm{T,\bar{b}}} \qquad \Rightarrow \qquad \alpha_\mathrm{S}^2(\mu^2_{\mathrm{tW^-}}) \simeq \alpha_\mathrm{S}(E^2_{\mathrm{T,t}})\alpha_\mathrm{S}(E^2_{\mathrm{T,\bar{b}}})$$

Global "interpolating scale"

$$\mu^2_{\mathrm{WWbb}} = \mu_{\mathrm{W^+b}}\, \mu_{\mathrm{W^-\bar{b}}} \quad \text{with} \quad \mu_{\mathrm{Wb}} = P_\mathrm{b}(p_{\mathrm{W,b}})\, E_{\mathrm{T,b}} + P_\mathrm{t}(p_{\mathrm{W,b}})\, E_{\mathrm{T,t}}$$

$\mathrm{g} \to \mathrm{b\bar{b}}$ and $\mathrm{t} \to \mathrm{Wb}$ **probabilities dictated by** respective **singularity structures**

$$\frac{P_\mathrm{b}}{P_\mathrm{t}} \propto \frac{\chi_\mathrm{b}}{\chi_\mathrm{t}} \qquad \text{with} \qquad \chi_\mathrm{b} = \frac{m_\mathrm{t}^2}{E^2_{\mathrm{T,b}}}, \quad \chi_\mathrm{t} = \frac{m_\mathrm{t}^4}{[(p_\mathrm{W}+p_\mathrm{b})^2 - m_\mathrm{t}^2]^2 + \Gamma_\mathrm{t}^2 m_\mathrm{t}^2},$$

and free constants fixed by **natural normalisation conditions**

$$P_\mathrm{b} + P_\mathrm{t} = 1, \qquad \text{and} \qquad \int \mathrm{d}\sigma^{\mathrm{FtW}}_{\mathrm{W^+W^-b\bar{b}}} = \int \mathrm{d}\Phi\, [1 - P_\mathrm{t}(\Phi) P_{\mathrm{\bar{t}}}(\Phi)]\, \frac{\mathrm{d}\sigma_{\mathrm{W^+W^-b\bar{b}}}}{\mathrm{d}\Phi}$$

Consistency of $t\bar{t}$ vs tW probability densities

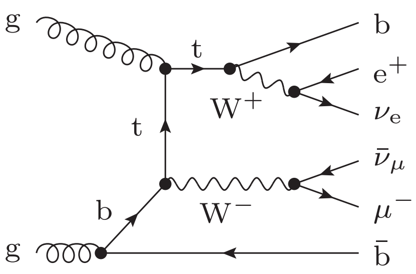
Check normalisation identity for more exclusive/differential observables

$$\int d\sigma^{\rm FtW}_{\rm W^+W^-b\bar{b}} = \int d\Phi \left[1 - P_{\rm t}(\Phi)P_{\bar{\rm t}}(\Phi)\right] \frac{d\sigma_{\rm W^+W^-b\bar{b}}}{d\Phi}$$

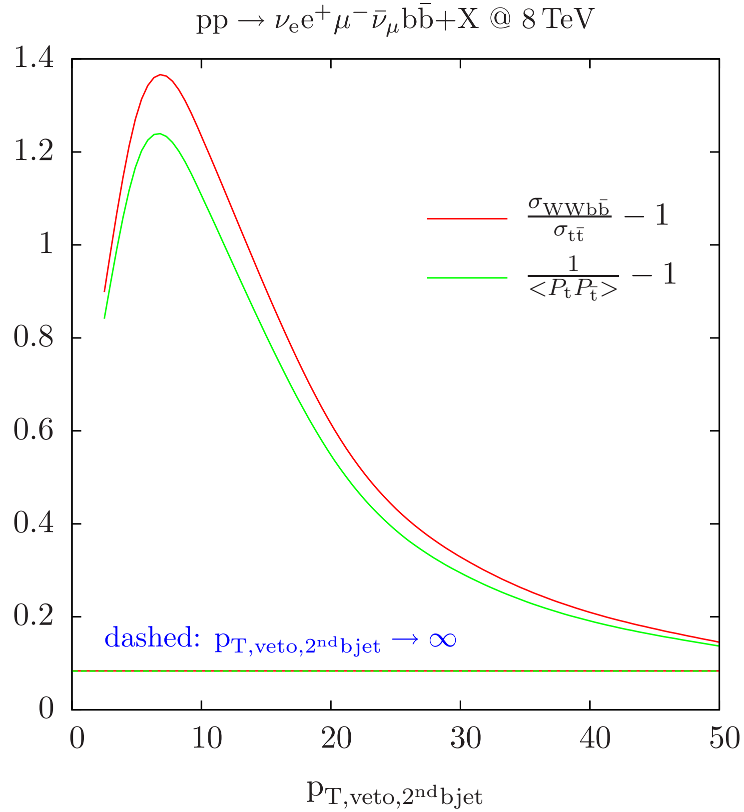
to verify if observed finite-top-width effects (computed via $\Gamma_{\rm t} \to 0$) are consistent with (pseudo)probability densities

Test dependence wrt veto on $2^{\rm nd}$ b-jet

- single-top Wt contribution strongly enhanced when $p_{\rm T,veto} \to 0$



- enhancement fairly well described by $P_{\rm t}(\Phi)$,$P_{\rm b}(\Phi)$ probability densities



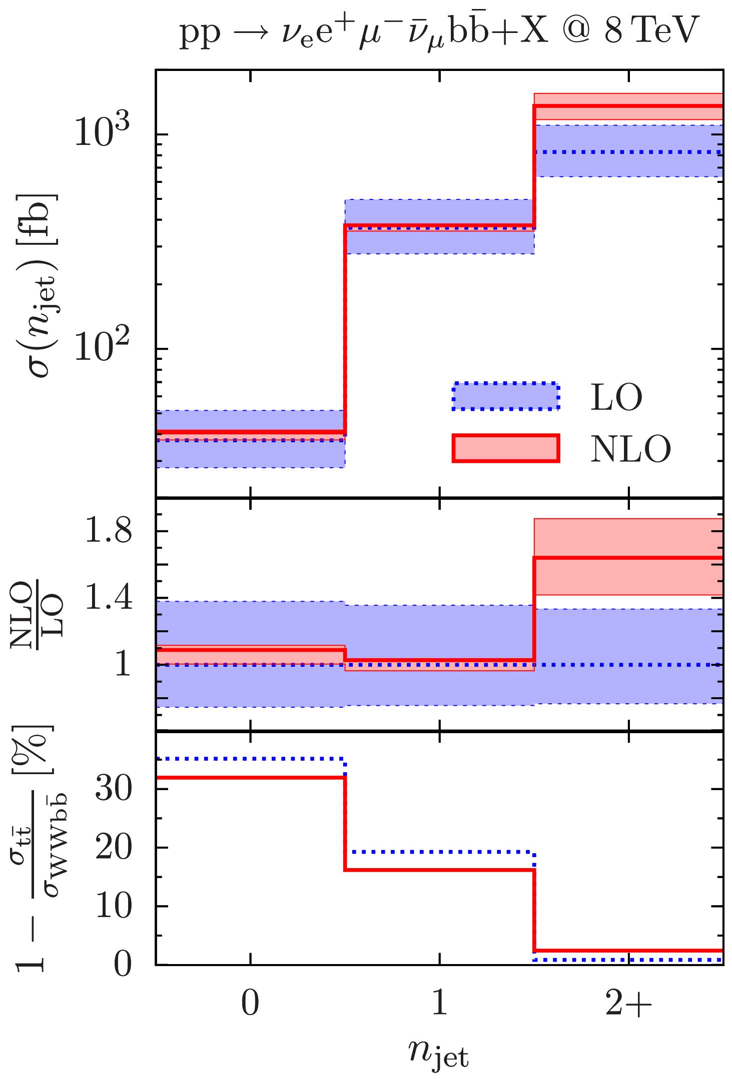
NLO and FtW effects in jet bins

Jet bins relevant for $t\bar{t}$-suppression and most interesting application of $m_b > 0$

- 40% inclusive NLO correction driven by 2-jet bin, with very stable 0/1-jet bins
- **only ∼10% NLO uncertainty in all bins!**
- FtW contribution bin-dependent (2% to 30%) and strongly enhanced in 0/1-jet bins!
- also FtW part perturbatively stable (not shown here)

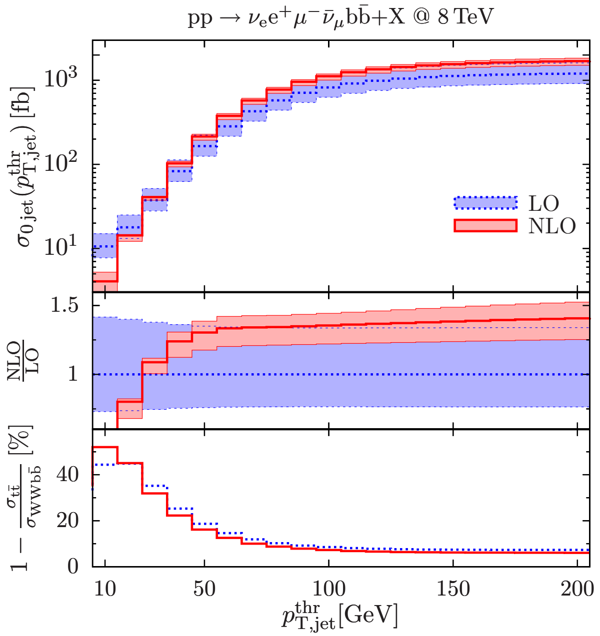
Success of "ad-hoc" scale choice

- but naive $\mu = m_t$ choice yields surprisingly similar stability in jet bins!
- "ad-hoc scale" should be superior for more exclusive observables...



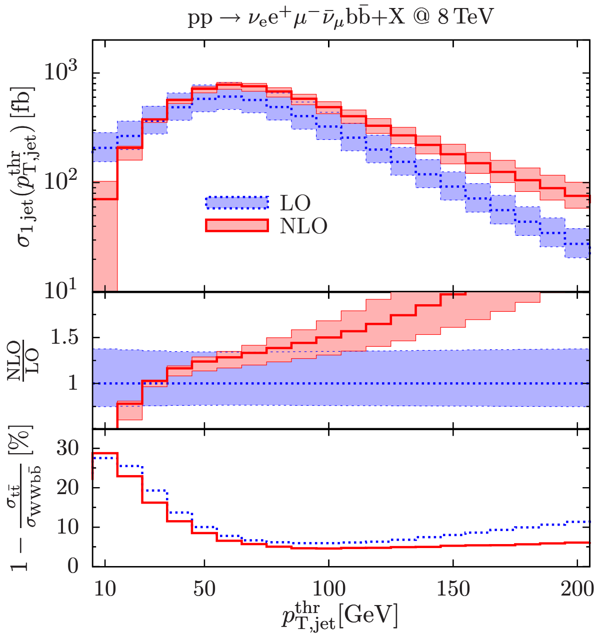
NLO(LO) 4F NNPDFSs, $p_{T,j} = 30$ GeV

Jet-Veto and Binning Effects



0-jet bin vs p_T-veto

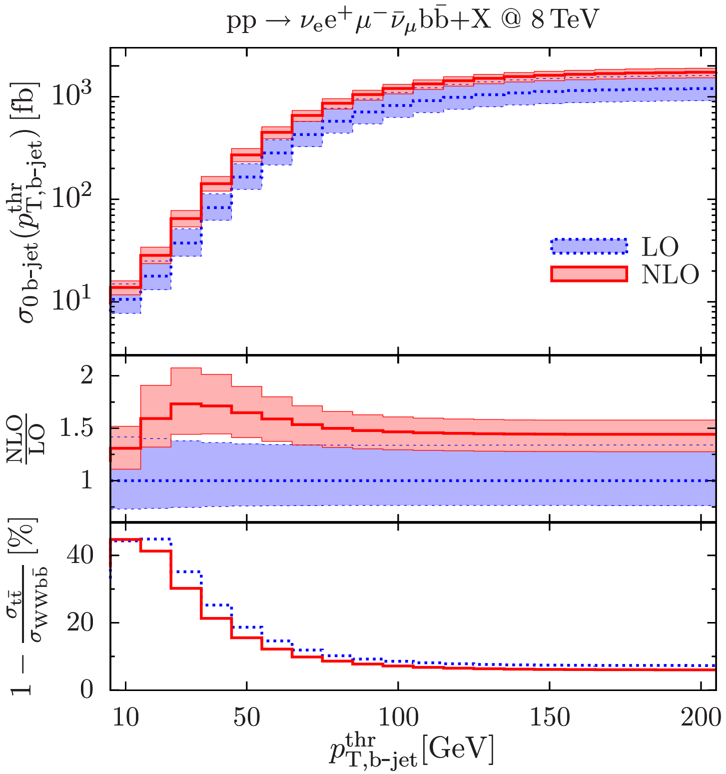
- smooth inclusive limit at large p_T and very strong p_T sensitivity below 50 GeV:
  - FtW effects increase up to 50%
  - K-factor falls very fast
- at low p_T IR singularity calls for NLO+PS matching
- typical veto $p_\mathrm{T} \sim 30$ GeV yields 98% suppression and still decent NLO stability ($K \sim 1$)

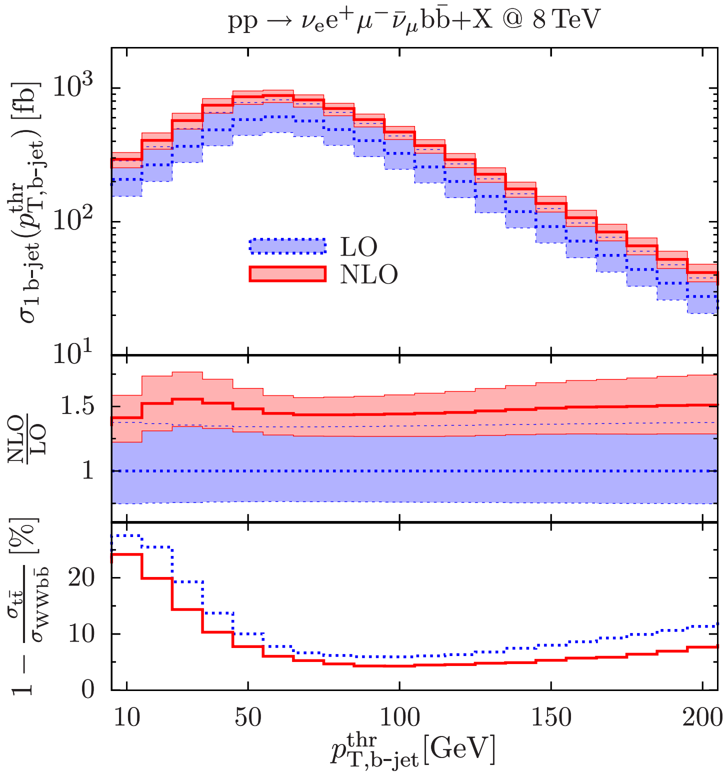


1-jet bin vs p_T threshold

- low p_T behaviour driven by veto on 2nd jet and analogous to 0-jet case
- high p_T region driven by 1st jet and NLO radiation dominates over b-jets from $\mathrm{W^+W^-b\bar{b}}$

B-Jet-Veto and Binning Effects

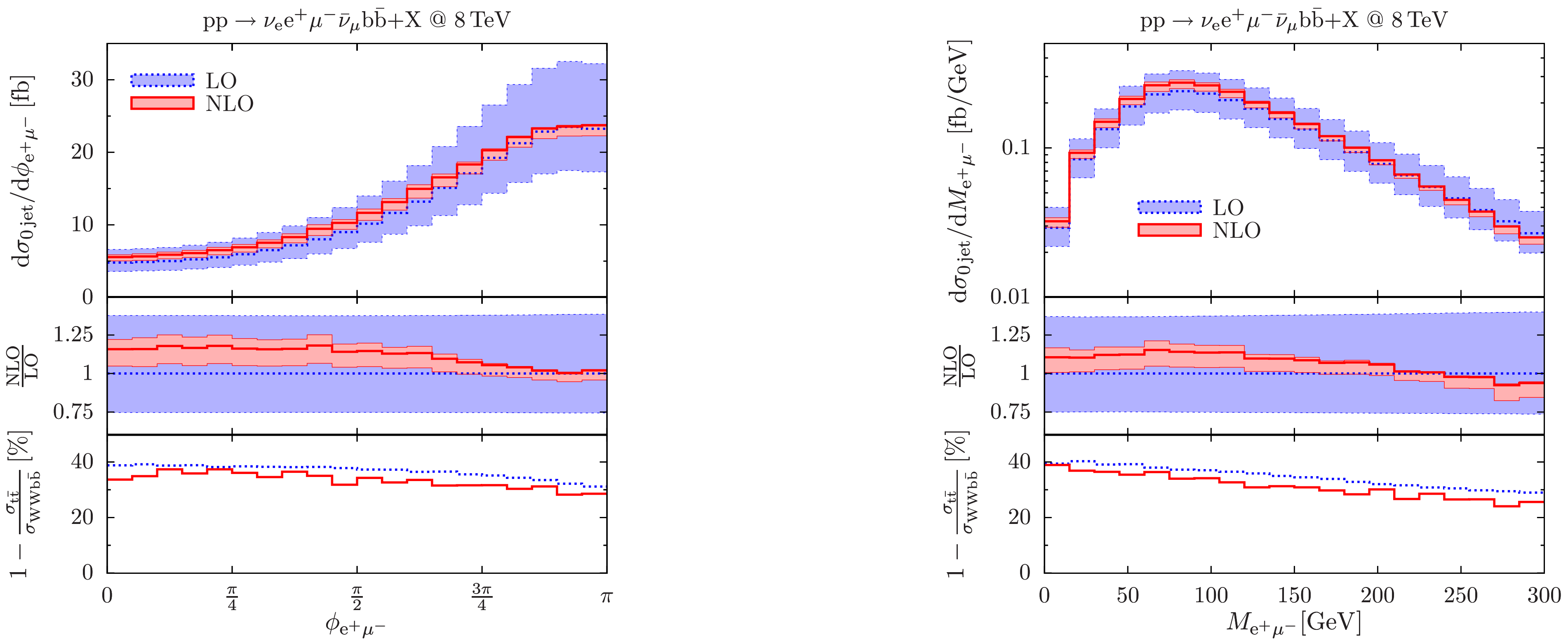




- NLO radiation doesn't change b-jet multiplicity ⇒ rather stable K-factor and uncertainties
- single-top and off-shell effects still enhanced at small b-jet p_{T}

In general: nontrivial interplay of NLO and off-shell/single-top effects

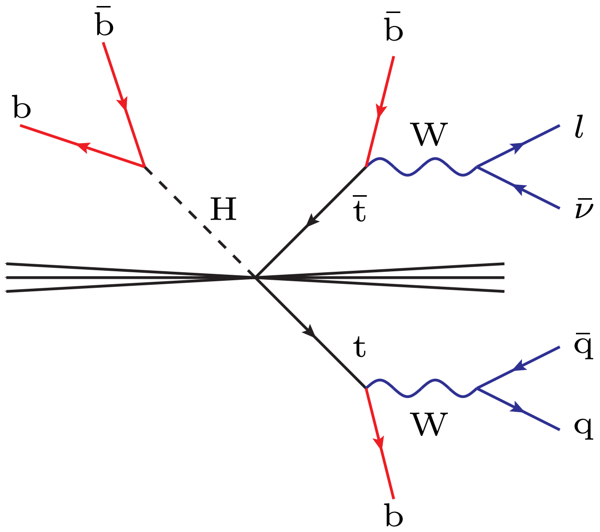
$t\bar{t}$ and Wt background to $H \to W^+W^-$ in 0-jet bin



- $\Delta\phi_{e^+\mu^-}$ and $M_{e^+\mu^-}$ distributions feature 10% NLO uncertainty
- significant (although moderate) NLO shape distortions
- 30–40% FtW contributions (nontrivial $t\bar{t}$/Wt mix)

(D) MC@NLO for 4F $t\bar{t}b\bar{b}$ production [Cascioli,Maieröfer,Moretti,S.P. ,Siegert '13]

$t\bar{t}H(b\bar{b})$ **Analyses at the LHC and Irreducible** $t\bar{t}b\bar{b}$ **Background**



- complicated $b\bar{b}b\bar{b}\ell\nu jj$ final state hampers $H \to b\bar{b}$ peak reconstruction
- signal still hidden in huge QCD background and search dominated by systematics
- theory uncertainty of irreducible $t\bar{t}b\bar{b}$ background crucial (normalisation in control region quite difficult)

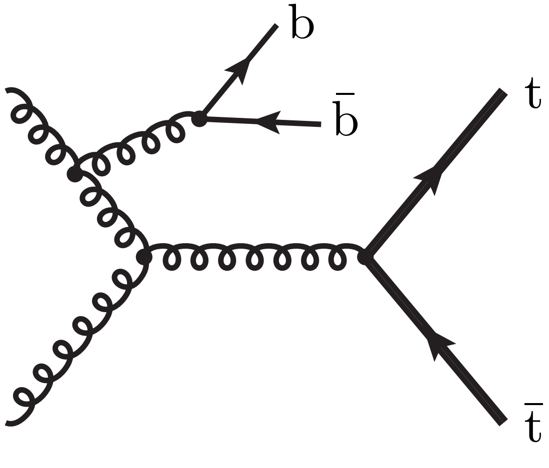
Theory predictions for $t\bar{t}b\bar{b}$ **background**

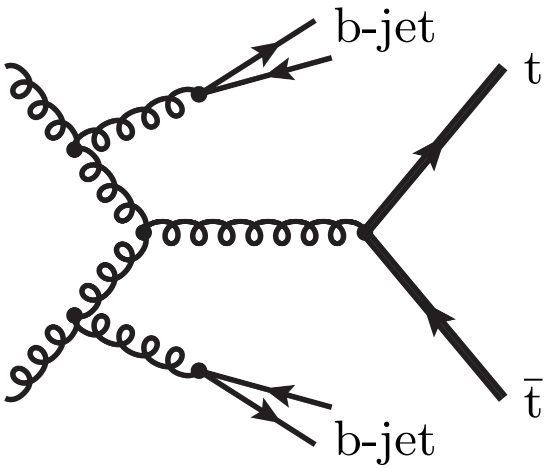
- NLO reduces scale uncertainty from 80% to 20–30% [Bredenstein, Denner, Dittmaier, S. P. '09/'10; Bevilacqua, Czakon, Papadopoulos, Pittau, Worek '09]
- application to ATLAS/CMS analyses requires matching to parton showers

→ POWHEG matching in **5F scheme** [Kardos, Trocsanyi '13]

→ Sherpa-MC@NLO matching in **4F scheme** [Cascioli, Maierhoefer, Moretti, S. P., Siegert '13]

NLO matching for $t\bar{t}b\bar{b}$ production in 5F vs 4F schemes





5F scheme ($m_b = 0$): $t\bar{t}b\bar{b}$ MEs cannot describe collinear $g \to b\bar{b}$ splittings

$\Rightarrow$ inclusive $t\bar{t}$+b-jets simulation requires $t\bar{t}g$+PS, i.e. $t\bar{t}+ \leq 2$ jets NLO merging

4F scheme ($m_b > 0$): $t\bar{t}b\bar{b}$ MEs cover full b-quark phase space

$\Rightarrow$ MC@NLO $t\bar{t}b\bar{b}$ sufficient for inclusive $t\bar{t}$+b-jets simulation

- access to **new** $t\bar{t} + 2$**b-jets production mechanism** wrt 5F scheme: double collinear $g \to b\bar{b}$ splittings (surprisingly important impact on $t\bar{t}H(b\bar{b})$ analysis!)

Sherpa Formulation [Höche, Krauss, Schönherr, Siegert '11] of MC@NLO Matching [Frixione, Webber '02]

MC@NLO matching (avoids double-counting of first emission)

$$\langle \mathcal{O} \rangle = \int d\Phi_B \Big[B(\Phi_B) + V(\Phi_B) + I(\Phi_B) \Big] U(t_0, \mu_Q^2)$$
$$+ \int d\Phi_R \Big[R(\Phi_R) - \sum_{ijk} D_{ijk}(\Phi_R) \theta(\mu_Q^2 - t) \Big] \mathcal{O}(\Phi_R).$$

Integrated CS dipole-subtraction terms

$$I(\Phi_B) = \sum_{ijk} \int d\Phi_{R|B} D_{ijk}(\Phi_R) \theta(\mu_Q^2 - t),$$

Sherpa shower based on CS dipoles (exact and automated colour treatment)

$$U(t_0, \mu_Q^2) = \Delta(t_0, \mu_Q^2) \mathcal{O}(\Phi_B) + \sum_{ijk} \int_{t_0}^{\mu_Q^2} d\Phi_{R|B} \frac{D_{ijk}(\Phi_R)}{B(\Phi_B)} \Delta(t, \mu_Q^2) \mathcal{O}(\Phi_R),$$

Resummation scale μ_Q (parton-shower starting scale) restricts shower to meaningful region and its variations provide systematic shower-uncertainty estimates

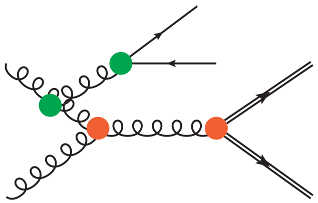
Scale choice and b-jet selections

Factorisation and Resummation scales (available phase space for QCD emission)

$$\mu_{\mathrm{F}} = \mu_Q = \frac{1}{2}(E_{\mathrm{T,t}} + E_{\mathrm{T,\bar{t}}})$$

Scale choice crucial due to $\alpha_S^4(\mu^2)$ dependence (80% LO variation)

- widely separated scales $m_{\mathrm{b}} \leq Q_{ij} \lesssim m_{\mathrm{t\bar{t}b\bar{b}}}$ can generate huge logs
- CKKW inspired scale adapts to b-jet p_{T} and guarantees good pert. convergence



$$\mu_{\mathrm{R}}^4 = E_{\mathrm{T,t}} E_{\mathrm{T,\bar{t}}} E_{\mathrm{T,b}} E_{\mathrm{T,\bar{b}}} \quad \Rightarrow \quad \alpha_S^4(\mu_{\mathrm{R}}^2) = \alpha_S(E_{\mathrm{T,t}}^2)\alpha_S(E_{\mathrm{T,\bar{t}}}^2)\alpha_S(E_{\mathrm{T,b}}^2)\alpha_S(E_{\mathrm{T,\bar{b}}}^2)$$

ttb, $ttbb$ and $ttbb_{100}$ analyses with stable tops

- ttb analysis ($N_{\mathrm{b}} \geq 1$)
- $ttbb$ analysis ($N_{\mathrm{b}} \geq 2$)
- $ttbb_{100}$ ($N_{\mathrm{b}} \geq 2$) analysis in the $\mathrm{t\bar{t}H(b\bar{b})}$ **signal region** $m_{\mathrm{bb}} > 100\,\mathrm{GeV}$

(N_{b}= number of QCD b-jets with $p_{\mathrm{T}} > 25\,\mathrm{GeV}$, $|\eta| < 2.5$ and at least one b-quark)

NLO corrections and uncertainties for ttb and $ttbb$ cross sections

	ttb	ttbb	ttbb($m_{\mathrm{bb}} > 100$)
σ_{LO}[fb]	$2547^{+71\%+14\%}_{-37\%-11\%}$	$463.9^{+66\%+15\%}_{-36\%-12\%}$	$123.7^{+62\%+17\%}_{-35\%-13\%}$
σ_{NLO}[fb]	$3192^{+33\%+4.6\%}_{-25\%-4.9\%}$	$557^{+28\%+5.6\%}_{-24\%-4.0\%}$	$141^{+25\%+8.6\%}_{-22\%-3.8\%}$
$\sigma_{\mathrm{NLO}}/\sigma_{\mathrm{LO}}$	1.25	1.20	1.14

MSTW2008 NLO(LO) 4F PDFs

Good perturbative convergence (also for ttb!)

- K-factors and uncertainties rather independent of selection
- +20% correction mainly from b-quark contribution to α_{S} running in 4F scheme ($K \simeq 1$ with 5F running)
- 20–30% residual uncertainty dominated by μ_{R} variations (1^{st} uncertainty)
- only 5-10% uncertainty from combined μ_{F} and μ_Q variations (2^{nd} uncertainty)

M@NLO corrections wrt NLO in ttb and $ttbb$ cross sections

	ttb	ttbb	ttbb($m_{\mathrm{bb}} > 100$)
$\sigma_{\mathrm{MC@NLO}}$[fb]	$3223^{+33\%+4.3\%}_{-25\%-2.5\%}$	$607^{+25\%+2.2\%}_{-22\%-2.8\%}$	$186^{+21\%+5.4\%}_{-20\%-4.7\%}$
$\sigma_{\mathrm{MC@NLO}}/\sigma_{\mathrm{NLO}}$	1.01	1.09	1.32
$\sigma^{\mathrm{2b}}_{\mathrm{MC@NLO}}$[fb]	3176	539	145
$\sigma^{\mathrm{2b}}_{\mathrm{MC@NLO}}/\sigma_{\mathrm{NLO}}$	0.99	0.97	1.03

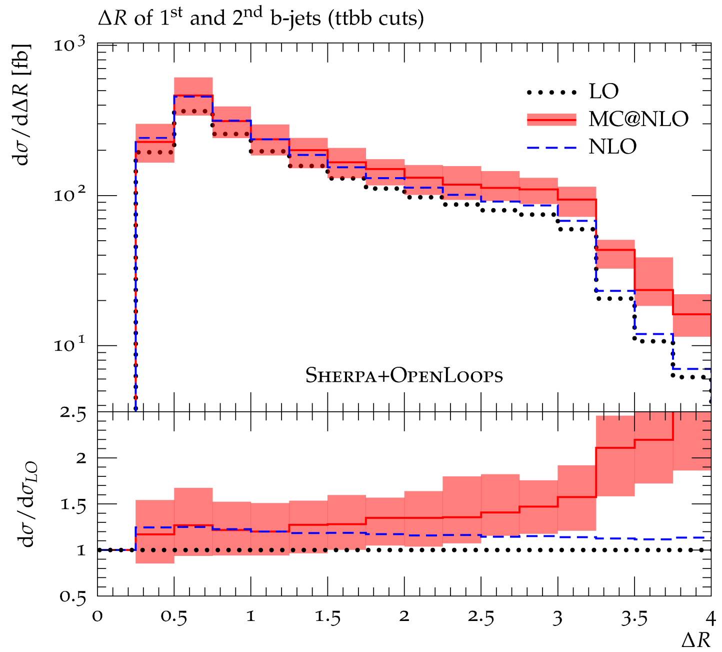
Nontrivial MC@NLO effects

- μ_{R}, μ_{F} and μ_Q uncertainties similar as for NLO
- negligible(moderate) MC@NLO/NLO differences with standard ttb($ttbb$) selections
- large MC@NLO effect ($\sim$30%) in Higgs-signal region of $ttbb$
- disappears in $\mathrm{MC@NLO}_{\mathrm{2b}}$, where $\mathrm{g} \to \mathrm{b\bar{b}}$ shower splittings are switched off (see more details in distributions)

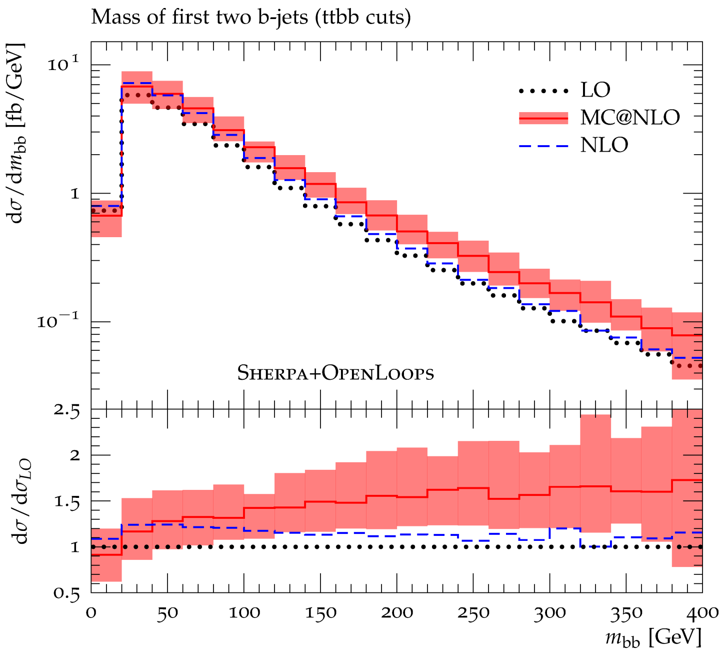
NLO and MC@NLO effects in distributions

$ttbb$ analysis ($N_b \geq 2$): b-jet correlations

$\Delta R_{\mathrm{b}_1\mathrm{b}_2}$



$m_{\mathrm{b}_1\mathrm{b}_2}$

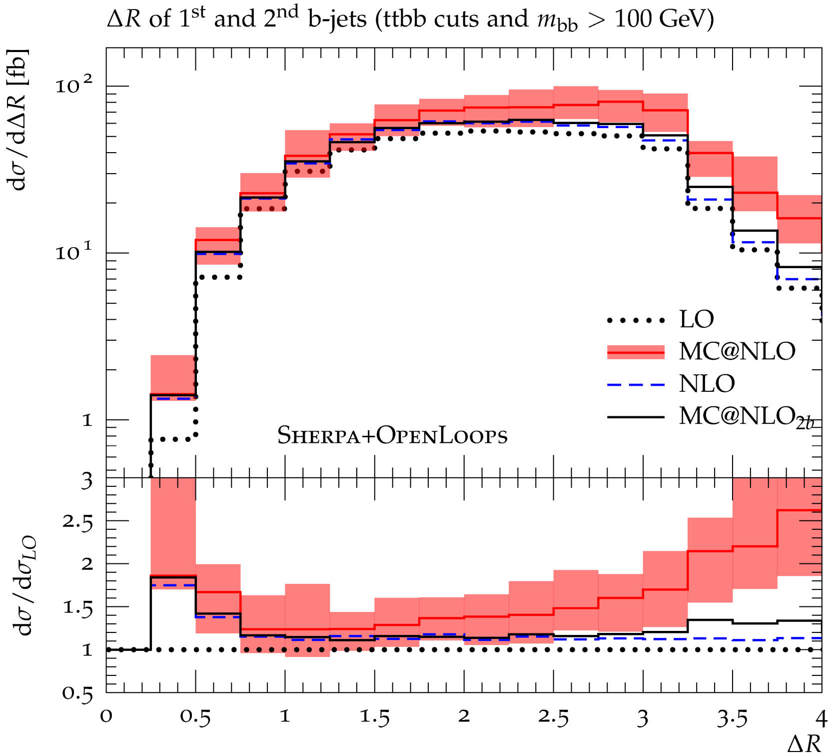


Unexpected behaviour

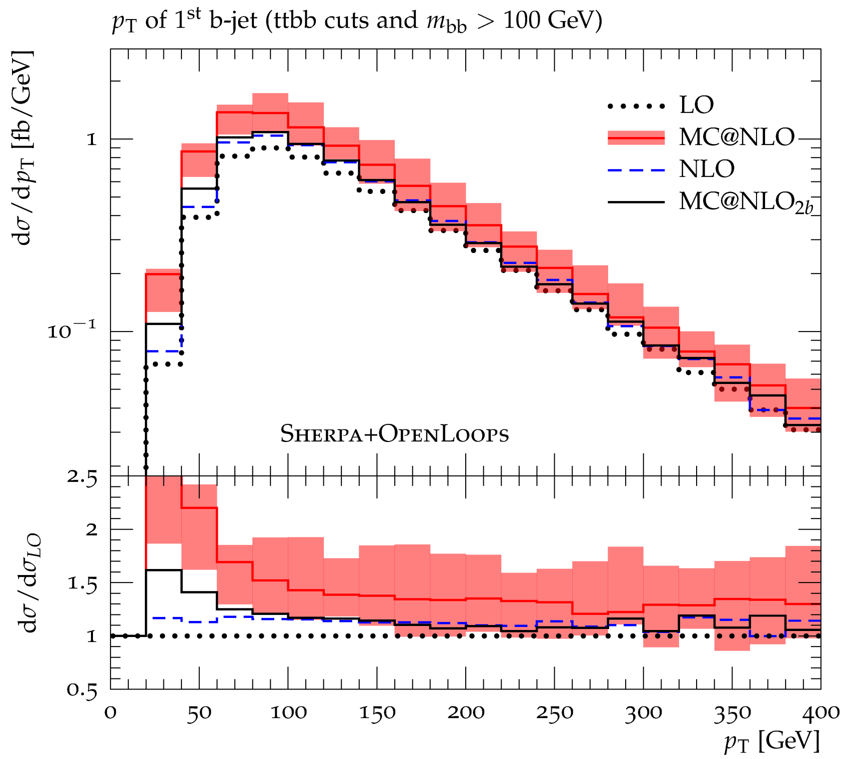
- NLO corrections quite flat
- **pronounced MC@NLO enhancement at large $\Delta R_{\mathrm{b}_1\mathrm{b}_2}$ and large $m_{\mathrm{b}_1\mathrm{b}_2}$**
- reaches 30–40% at $m_{\mathrm{b}_1\mathrm{b}_2} \sim 125\,\mathrm{GeV}$ and **largely exceeds** $\mathrm{t\bar{t}H(b\bar{b})}$ **signal!**

ttbb **analysis** $(N_b \geq 2)$ **with** $m_{\mathrm{b_1 b_2}} > 100\,\mathrm{GeV}$**:** b-jet observables

$\Delta R_{\mathrm{b_1 b_2}}$

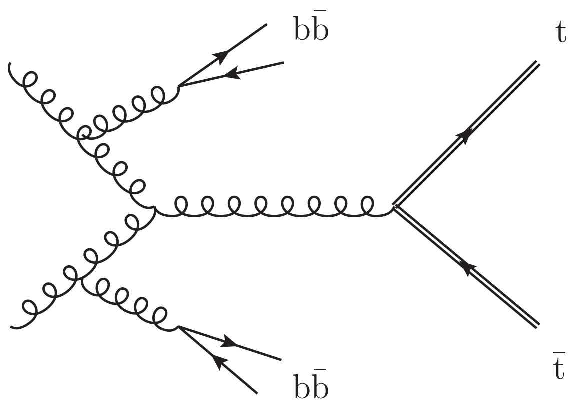


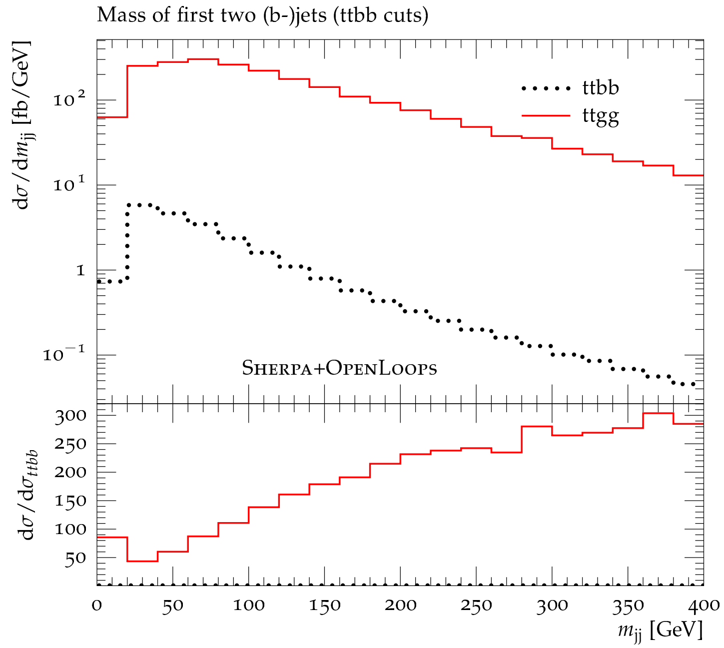
$p_{\mathrm{T,b_1}}$



MC@NLO excess at large m_{bb} **from back-to-back soft jets**

- factor-2 enhancement at $\Delta R \sim \pi$ and at small p_{T}
- disappears almost completely in MC@NLO$_{\mathrm{2b}}$ where $\mathrm{g} \to \mathrm{b\bar{b}}$ splittings are switched off in the parton shower (double $\mathrm{g} \to \mathrm{b\bar{b}}$ splittings "smoking gun")

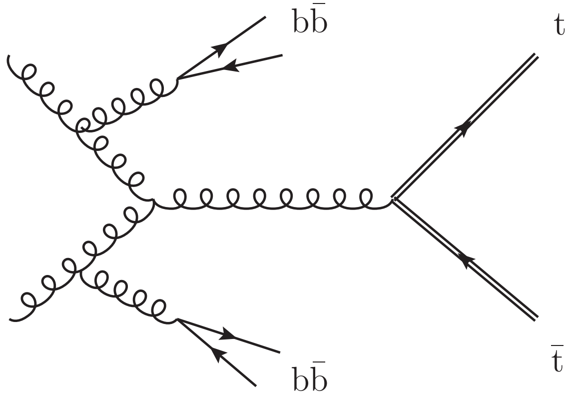




MC@NLO enhancement consistent with double $g \to b\bar{b}$ splittings mechanism

- "double splittings" kinematically favoured at large m_{bb} since $t\bar{t}gg/t\bar{t}b\bar{b}$ ratio grows and $g \to b\bar{b}$ splitting probability does not decrease at large m_{gg}
- emission of parent gluons is strongly enhanced at small p_T due to double (soft-collinear) singularity associated to IS gluon emission $\Rightarrow$ at large invariant mass the di-jet system tends to have the smallest possible p_T and $\Delta R \sim \pi$
- kinematic reconstruction of double $g \to b\bar{b}$ splitting nontrivial since typically $\Delta R_{b\bar{b}} > 0.4$ and one of the b-quarks can be outside acceptance

Implications of (double) $g \to b\bar{b}$ splitting contributions



Double splittings change conventional hard-scattering picture

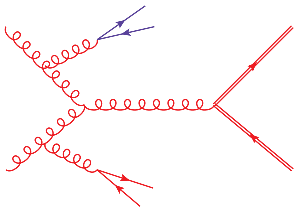
- this kind of contributions have always been present in $t\bar{t}$+jets LO merged samples
- however, their large impact on the $t\bar{t}H(b\bar{b})$ signal region is surprising and does not fit into the conventional hard-scattering picture of $t\bar{t}b\bar{b}$ production based on a *single* and *non-collinear* $b\bar{b}$ pair

Implications for theory systematics in $t\bar{t}$+HF

- matching to shower essential (4F $t\bar{t}b\bar{b}$ NLO matching or 5F $t\bar{t}$+jets NLO merging)
- MC@NLO $t\bar{t}b\bar{b}$ simulation provides NLO accuracy for $t\bar{t}$+2 b-jets with hard b-quark jets: NLO or LO+PS accuracy for "double-splittings"?

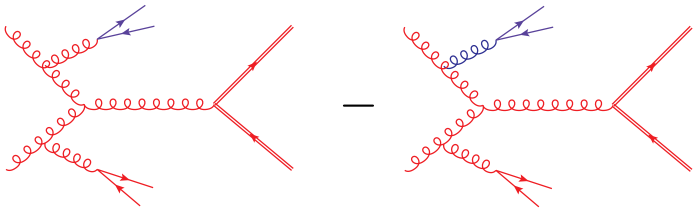
Accuracy of “double splittings” in MC@NLO *ttbb* simulation

Naive picture



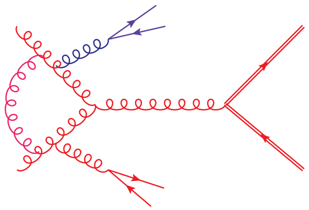
real-emission $\mathrm{t\bar{t}b\bar{b}g}$ MEs plus $\mathrm{g \to b\bar{b}}$ shower splitting
$\Rightarrow$ only LO+PS accuracy as in usual LO merging

Correct MC@NLO picture: interplay of three different contributions



$\mathrm{t\bar{t}b\bar{b}g}$ MEs plus PS $\mathrm{g \to b\bar{b}}$ emission

- LO $\mathrm{t\bar{t}b\bar{b}g}$ uncertainty ~100% at large p_T
- largely cancelled by PS-matching at small p_T

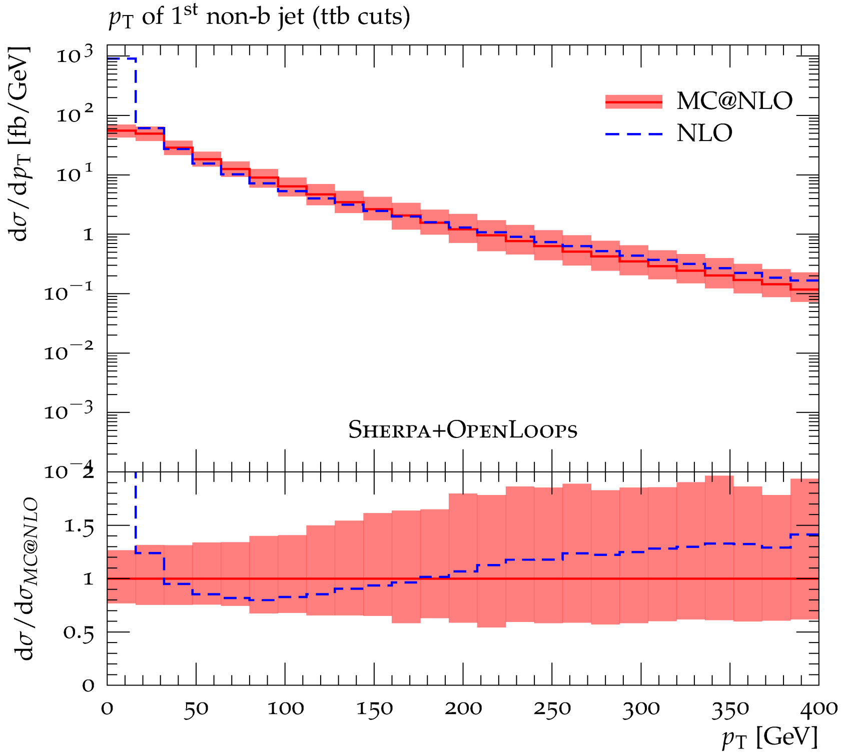


$\mathrm{t\bar{t}b\bar{b}}$ MEs plus PS gluon and $\mathrm{g \to b\bar{b}}$ emissions

- dominates at small p_T
- NLO $\mathrm{t\bar{t}b\bar{b}}$ accuracy ~25%

Well reflected in scale uncertainty of 1$^{\mathrm{st}}$ light-jet emission on top of $\mathrm{t\bar{t}b\bar{b}}$...

ttb **analysis** ($N_b \geq 1$)**:** 1^{st} light-jet p_{T} distribution (responsible for double splittings)



MC@NLO vs NLO

- Sudakov damping of NLO IR singularity at $p_{\text{T}} \to 0$
- 30% NLO excess in the hard tail (probably due to dynamic μ_Q, multi-jet final state, unresolved b-quark)

MC@NLO scale uncertainty

- LO-like uncertainty (∼100%) in the tail irrelevant for $\mathrm{t\bar{t}H(b\bar{b})}$
- NLO-like accuracy (∼30%) up to 70 GeV

$\Rightarrow$ **NLO-like accuracy in the region relevant for** $\mathrm{t\bar{t}H(b\bar{b})}$

Conclusions

OpenLoops

- handles $2 \to 2, 3, 4$ SM process at NLO QCD very efficiently
- well tested, working for nontrivial LHC studies, ready for publication

Examples of first applications ($\mathrm{W^+W^-b\bar{b}}$ and $\mathrm{t\bar{t}b\bar{b}}$)

- $m_\mathrm{b} > 0$ and NLO matching give access to new important physics ingredients (single-top, double splittings) and crucial for applicability to exp analysis
- ~ 4 years after first NLO papers (2009, 2011) and not yet the end of the story (top decays in $\mathrm{t\bar{t}b\bar{b}}$, NLO matching for $\mathrm{W^+W^-b\bar{b}}$, nontrivial pheno applications like m_t measurements,...)

Lesson

- NLO $\mathrm{t\bar{t}}$ still very active business 25 years after first pioneering result
- NLO automation is just moving the first (very promising) steps
- the very wide applicability range of NLO tools and high relevance for the LHC will stimulate further exciting progress

BACKUP SLIDES

$W^+W^-b\bar{b}$ cross section in generic-jet bins

	μ_0	σ[fb]	σ_0[fb]	σ_1[fb]	σ_{2+}[fb]
LO	μ_{WWbb}	$1232^{+34\%}_{-24\%}$	$37^{+38\%}_{-25\%}$	$367^{+36\%}_{-24\%}$	$828^{+33\%}_{-23\%}$
NLO	μ_{WWbb}	$1777^{+10\%}_{-12\%}$	$41^{+3\%}_{-8\%}$	$377^{+1\%}_{-6\%}$	$1359^{+14\%}_{-14\%}$
K	μ_{WWbb}	1.44	1.09	1.03	1.64
LO	m_{t}	$1317^{+35\%}_{-24\%}$	$35^{+37\%}_{-25\%}$	$373^{+36\%}_{-24\%}$	$909^{+35\%}_{-24\%}$
NLO	m_{t}	$1817^{+8\%}_{-11\%}$	$40^{+4\%}_{-8\%}$	$372^{+1\%}_{-8\%}$	$1405^{+13\%}_{-13\%}$
K	m_{t}	1.38	1.14	1.00	1.55
	μ_0	σ^{FtW}[fb]	σ_0^{FtW}[fb]	σ_1^{FtW}[fb]	$\sigma_{2+}^{\mathrm{FtW}}$[fb]
LO	μ_{WWbb}	$91^{+41\%}_{-27\%}$	$13^{+42\%}_{-27\%}$	$71^{+40\%}_{-27\%}$	$7^{+45\%}_{-29\%}$
NLO	μ_{WWbb}	$107^{+6\%}_{-11\%}$	$13^{+1\%}_{-7\%}$	$61^{+2\%}_{-16\%}$	$33^{+51\%}_{-31\%}$
K	μ_{WWbb}	1.18	0.99	0.86	4.70
LO	m_{t}	$63^{+36\%}_{-25\%}$	$8^{+36\%}_{-25\%}$	$49^{+36\%}_{-24\%}$	$6^{+46\%}_{-29\%}$
NLO	m_{t}	$100^{+17\%}_{-16\%}$	$13^{+14\%}_{-14\%}$	$65^{+9\%}_{-12\%}$	$23^{+42\%}_{-28\%}$
K	m_{t}	1.58	1.47	1.32	3.89

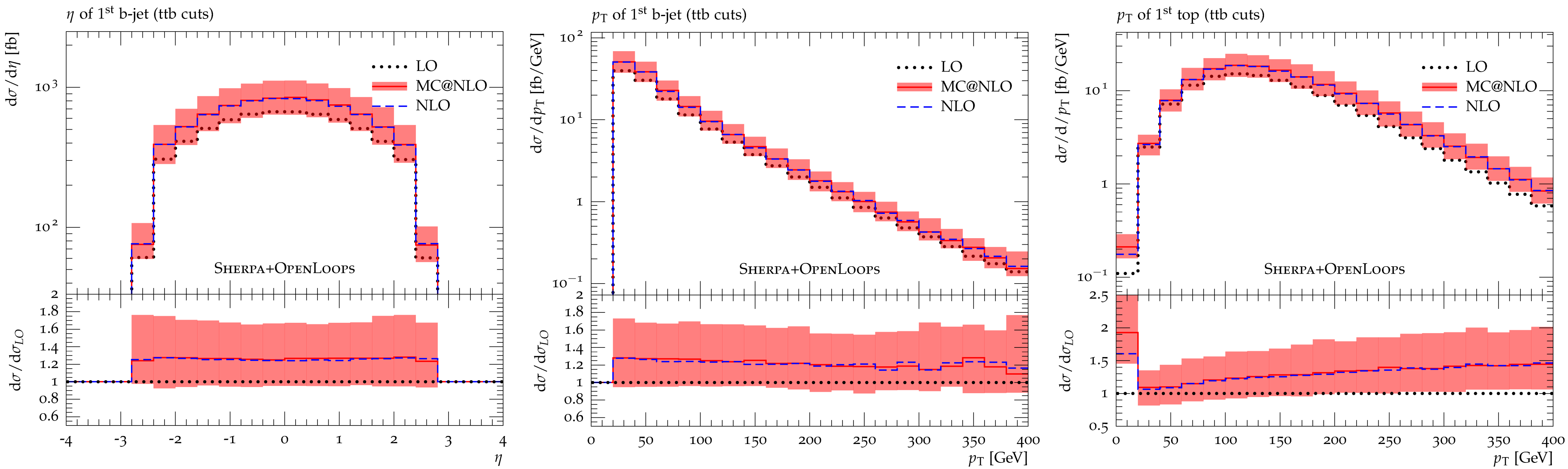
$W^+W^-b\bar{b}$ cross section in b-jet bins

	μ_0	σ[fb]	σ_0[fb]	σ_1[fb]	σ_{2+}[fb]
LO	μ_{WWbb}	$1232^{+34\%}_{-24\%}$	$37^{+38\%}_{-25\%}$	$367^{+36\%}_{-24\%}$	$828^{+33\%}_{-23\%}$
NLO	μ_{WWbb}	$1777^{+10\%}_{-12\%}$	$65^{+20\%}_{-17\%}$	$571^{+14\%}_{-14\%}$	$1140^{+7\%}_{-10\%}$
K	μ_{WWbb}	1.44	1.73	1.56	1.38
LO	m_t	$1317^{+35\%}_{-24\%}$	$35^{+37\%}_{-25\%}$	$373^{+36\%}_{-24\%}$	$909^{+35\%}_{-24\%}$
NLO	m_t	$1817^{+8\%}_{-11\%}$	$63^{+20\%}_{-17\%}$	$584^{+14\%}_{-14\%}$	$1170^{+5\%}_{-9\%}$
K	m_t	1.38	1.80	1.56	1.29
	μ_0	σ^{FtW}[fb]	σ_0^{FtW}[fb]	σ_1^{FtW}[fb]	σ_{2+}^{FtW}[fb]
LO	μ_{WWbb}	$91^{+41\%}_{-27\%}$	$13^{+42\%}_{-27\%}$	$71^{+40\%}_{-27\%}$	$7^{+45\%}_{-29\%}$
NLO	μ_{WWbb}	$107^{+6\%}_{-11\%}$	$20^{+18\%}_{-17\%}$	$82^{+4\%}_{-10\%}$	$5^{+2\%}_{-10\%}$
K	μ_{WWbb}	1.18	1.49	1.16	0.77
LO	m_t	$63^{+36\%}_{-25\%}$	$8^{+36\%}_{-25\%}$	$49^{+36\%}_{-24\%}$	$6^{+46\%}_{-29\%}$
NLO	m_t	$100^{+17\%}_{-16\%}$	$16^{+22\%}_{-18\%}$	$77^{+16\%}_{-15\%}$	$6^{+12\%}_{-16\%}$
K	m_t	1.58	1.89	1.58	1.10

NLO and MC@NLO effects in distributions

ttb **analysis ($N_b \geq 1$):** b-jet and top-quark distributions

η_{b_1} $\qquad$ p_{T,b_1} $\qquad$ p_{T,t_1}

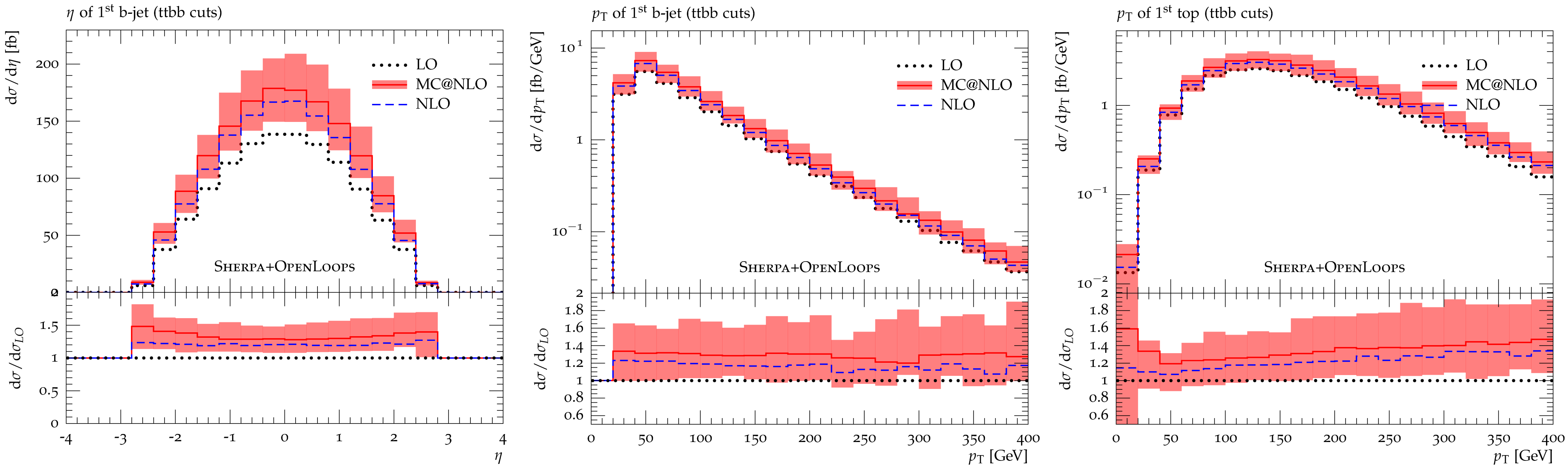


Reliable perturbative prediction

- shape of 1^{st} b-jet very stable wrt NLO corrections (thanks to dynamic scale!)
- shape of 1^{st} top receives significant (∼25%) NLO correction
- **excellent MC@NLO vs NLO agreement**

$ttbb$ **analysis** ($N_b \geq 2$): b-jet and top-quark distributions

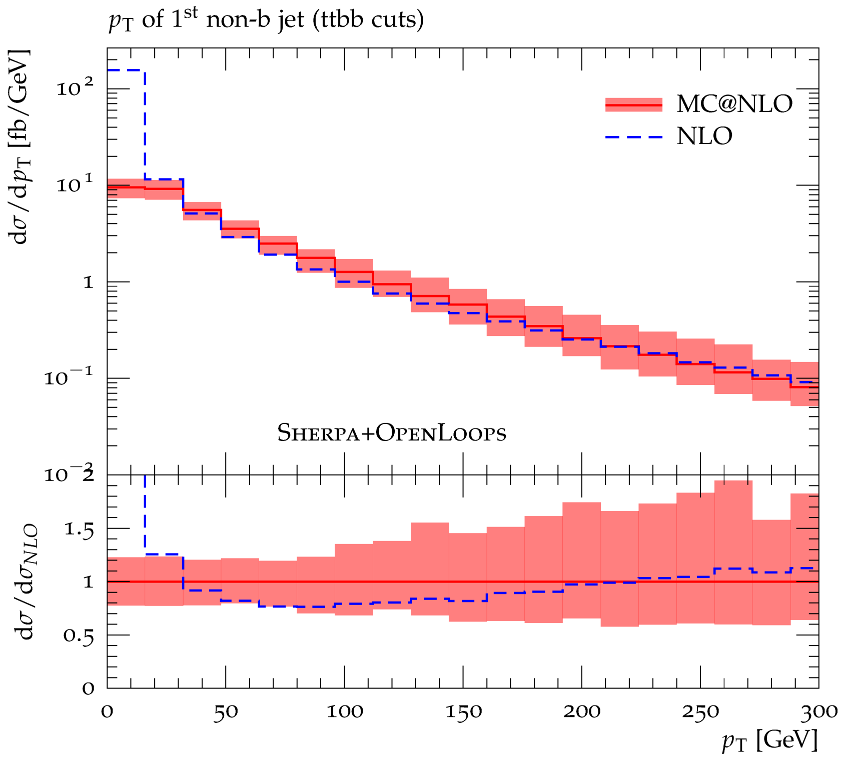
η_{b_1} $\qquad$ p_{T,b_1} $\qquad$ p_{T,t_1}



Similarly good stability as for ***ttb*** **analysis**

- apart from moderate MC@NLO excess wrt NLO
- resulting distortions of b-jet and top distributions very mild

ttbb **analysis** ($N_b \geq 2$)**:** 1^{st} light-jet p_{T} distribution



MC@NLO vs NLO

- in good (5%) agreement in the tail
- Sudakov damping of NLO IR singularity at $p_{\mathrm{T}} \to 0$
- $\sim$25% deviation at intermediate p_{T} consistent with expected NNLO effect

MC@NLO scale uncertainty

- LO-like uncertainty ($\sim$100%) in the tail irrelevant for $t\bar{t}H(b\bar{b})$
- NLO-like accuracy ($\sim$25%) up to 100 GeV