Beta-beam and New Physics

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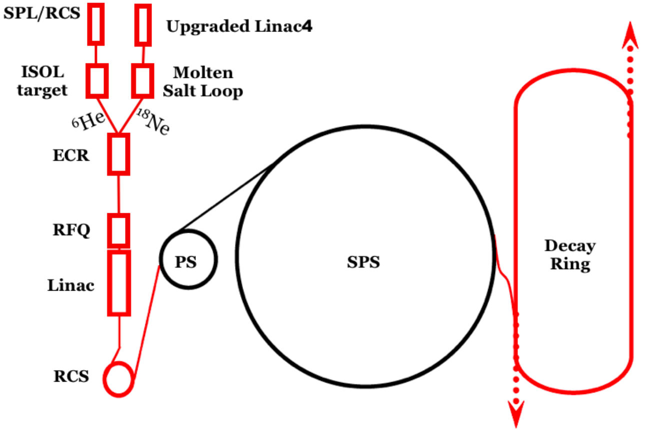
Outline

1. Introduction: the beta–beam
2. New Physics and neutron beta decay
3. New Physics and beta–beam

The beta–beam

Originally proposed in: P. Zucchelli, Phys. Lett. B **532** (2002) 166.



Picture from: C. Hansen, arXiv:1109.1956

Decay Ring length ≈ SPS length (≈ 7 km), straight sections: 2 × 36% of total length.

$$ {}^{6}_{2}\mathrm{He} \rightarrow {}^{6}_{3}\mathrm{Li} + e^{-} + \bar{\nu}_e \,, \;\; T_{1/2} \approx 0.81\,\mathrm{s}\,, \;\; Q_\beta \approx 3.51\,\mathrm{MeV} $$

"The PS collects 20 RCS bunches one by one. It will be 1.9 seconds between the first and last injection. There will therefore be intensity differences in the bunches when all 20 bunches are injected to the PS due to radioactive decay"

from: C. Hansen, arXiv:1109.1956

$$ \langle E_\nu^{CM} \rangle = 1.94\,\mathrm{MeV}\,, \;\; \langle E_\nu^{LAB} \rangle \approx 2\gamma \langle E_\nu^{CM} \rangle = 388\,\mathrm{MeV}\,, (\gamma = 100)\,, $$

The distance CERN – Fréjus Underground Laboratory ($130\,\mathrm{km}$) is close to the first $\nu_e \rightarrow \nu_\mu$ oscillation maximum.

$^{18}\mathrm{Ne}$: ν_e emitter with $T_{1/2} \approx 1.67\,\mathrm{s}$, $Q_\beta \approx 3.41\,\mathrm{MeV}$ (main decay fraction)

General Hamiltonian

P. Herczeg, Prog. Part. Nucl. Phys. **46** (2001) 413.

$$\mathcal{H} = 4 \sum_{i=1}^{3} \sum_{\ell=e,\mu,\tau} \sum_{k,l=L,R} \Big\{ a_{kl}\, U^k_{\ell i}\, \bar{\ell} \gamma_\mu P_k \nu_i\, \bar{u} \gamma^\mu P_l d$$

$$+ \; A_{kl}\, U^k_{\ell i}\, \bar{\ell} P_k \nu_i\, \bar{u} P_l d \; + \; \alpha_{kk}\, U^k_{\ell i}\, \bar{\ell} \frac{\sigma_{\mu\nu}}{\sqrt{2}} P_k \nu_i\, \bar{u} \frac{\sigma^{\mu\nu}}{\sqrt{2}} P_k d \Big\} + \text{H.c.},$$

$$P_L = \frac{1}{2}(1 - \gamma_5) \; , P_R = \frac{1}{2}(1 + \gamma_5) \; , \sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu]$$

- U^L and U^R are 3×3 unitary mixing matrices;
- we assume that a_{kl}, A_{kl}, α_{kk} are real parameters.

Standard Model: $a_{LL} = \frac{\mathrm{V}_{ud} G_F}{\sqrt{2}}$ and all other 0.

The free neutron beta decay

In order to calculate amplitudes for

$$n \longrightarrow p + e^- + \bar{\nu}_i \,,$$

we need to know

$$\langle p(p_p, \lambda_p)|\bar{u}(x) O_i d(x)|n(p_n, \lambda_n)\rangle = \bar{\Psi}_p(x, p_p, \lambda_p) H_i(p_p, p_n) \Psi_n(x, p_n, \lambda_n) \,,$$

where $i = S,\ P,\ V,\ A,\ T$, while O_i are given by

$$O_S = 1\,,\ \ O_P = \gamma_5\,,\ \ O_V = \gamma_\mu\,,\ \ O_A = \gamma_\mu \gamma_5\,,\ \ O_T = \sigma_{\mu\nu}\,.$$

The wave functions are ($j = p,\ n$)

$$\Psi_j(x, p_j, \lambda_j) = u_j(p_j, \lambda_j) e^{-ip_j x}\,.$$

Form factors

S. Weinberg, Phys. Rev. 112 (1958) 1375

$$H_S = g_S(q^2)\,,$$
$$H_P = g_P(q^2)\,\gamma_5\,,$$
$$H_V = F_1(q^2)\,\gamma_\mu + \frac{iF_2(q^2)}{2m_N}\,\sigma_{\mu\nu}q^\nu + \frac{F_3(q^2)}{m_N}\,q_\mu\,,$$
$$H_A = G_A(q^2)\,\gamma_\mu\gamma_5 + \frac{G_P(q^2)}{m_N}\,q_\mu\gamma_5 + \frac{G_3(q^2)}{m_N}\,P_\mu\gamma_5\,,$$
$$H_T = g_T(q^2)\,\sigma_{\mu\nu} + \frac{ig_T^{(1)}(q^2)}{m_N}\,(q_\mu\gamma_\nu - q_\nu\gamma_\mu)$$
$$+ \frac{ig_T^{(2)}(q^2)}{m_N^2}\,(q_\mu P_\nu - q_\nu P_\mu) + \frac{ig_T^{(3)}(q^2)}{m_N}\,(\gamma_\mu q^\alpha\gamma_\alpha\gamma_\nu - \gamma_\nu q^\alpha\gamma_\alpha\gamma_\mu)\,,$$

$m_N = (m_n + m_p)/2$, $P = p_n + p_p$ and $q = p_n - p_p$.

Form factors

S. Weinberg, Phys. Rev. 112 (1958) 1375

- All form factors are real functions as strong interactions are time reversal invariant.
- If $m_u = m_d$ then the QCD Lagrangian is invariant under the isospin symmetry

$$\begin{pmatrix} u(x) \\ d(x) \end{pmatrix} \to \exp\left(-i\boldsymbol{\theta}\cdot\boldsymbol{\sigma}/2\right)\begin{pmatrix} u(x) \\ d(x) \end{pmatrix},$$

where $\boldsymbol{\theta} = (\theta_1, \theta_2, \theta_3)$ and $\boldsymbol{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$. If we further set $m_n = m_p$ then

$$F_3(q^2) \equiv 0\,, \quad G_3(q^2) \equiv 0\,, \quad g_T^{(3)}(q^2) \equiv 0\,.$$

Scattering of antineutrinos on free protons

The matrix elements needed for ($\ell = e, \mu, \tau$)

$$\bar{\nu}_i + p \to \ell^+ + n$$

can be obtained from

$$\langle n(p_n, \lambda_n)|\bar{d}(x)O_i u(x)|p(p_p, \lambda_p)\rangle = \delta_i \langle p(p_p, \lambda_p)|\bar{u}(x)O_i d(x)|n(p_n, \lambda_n)\rangle^* ,$$

where $\delta_{S,V,A,T} = 1$ and $\delta_P = -1$.

Form factors in the SM ($Q^2 = -q^2$)

$$F_1(Q^2) = \frac{[G_E^p(Q^2) - G_E^n(Q^2)] - \frac{Q^2}{4m_N^2}[G_M^p(Q^2) - G_M^n(Q^2)]}{1 + \frac{Q^2}{4m_N^2}},$$

$$F_2(Q^2) = \frac{[G_M^p(Q^2) - G_M^n(Q^2)] - [G_E^p(Q^2) - G_E^n(Q^2)]}{1 + \frac{Q^2}{4m_N^2}}.$$

$$G_E^p(Q^2) = G_D(Q^2)\,, \qquad G_E^n(Q^2) = 0\,,$$

$$G_M^p(Q^2) = \frac{\mu_p}{\mu_N} G_D(Q^2)\,, \qquad G_M^n(Q^2) = \frac{\mu_n}{\mu_N} G_D(Q^2)\,,$$

where $\mu_p \approx 2.793\,\mu_N$ and $\mu_n \approx -1.913\,\mu_N$ are magnetic moments, μ_N is the nuclear magneton,

$$G_D(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{M_V^2}\right)^2},$$

where $M_V \approx 0.84\,\mathrm{GeV}$ (fitted from electron scattering data).

Form factors in the SM

The axial form factor:

$$G_A(Q^2) = \frac{g_A}{\left(1 + \frac{Q^2}{M_A^2}\right)^2}\,.$$

For $g_A = 1.2673$ we have $M_A = 1.026 \pm 0.021\,\mathrm{GeV}$.

The pseudoscalar form factor:

$$G_P(Q^2) = G_A(Q^2)\frac{2m_N^2}{m_\pi^2 + Q^2}\,.$$

Correlation coefficients

J. D. Jackson, S. B. Treiman, H. W. Wyld, Jr., Phys. Rev. 106 (1957a) 517
J. Holeczek, M. Ochman, E. Stephan, M. Zralek, Acta Phys. Polon. B 42 (2011) 2493

J. Holeczek, M. Ochman, E. Stephan, M. Zralek, arXiv:1303.5295v1

$$\frac{d\Gamma}{dE_e d\Omega_e d\Omega_\nu} \sim \frac{|\mathbf{p}_e| E_e E_\nu^2}{(2\pi)^5} \Bigg\{ 1 + a \frac{\mathbf{p}_e \cdot \mathbf{p}_\nu}{E_e E_\nu} + b \frac{m_e}{E_e} + \boldsymbol{\lambda}_n \cdot \left[A \frac{\mathbf{p}_e}{E_e} + B \frac{\mathbf{p}_\nu}{E_\nu} + D \frac{\mathbf{p}_e \times \mathbf{p}_\nu}{E_e E_\nu} \right] \Bigg\},$$

$$B = B_0 + b_\nu \frac{m_e}{E_e} \,,$$

in Standard Model: $b \equiv 0$ and $b_\nu \equiv 0$.

$b \equiv 0$ and $b_\nu \equiv 0$: $s_L \equiv 0$ and $T_{LL} \equiv 0$ as well as no mixing between vactors and scalars/tensors.

PDG 2012: $D = (-1.2 \pm 2.0) \times 10^{-4}$

s_L and α_{LL}

T. Bhattacharya et al., Phys. Rev. D 85 (2012) 054512

From superallowed Fermi nuclear transitions:

$$-1.0 \times 10^{-3} < s_L < 3.2 \times 10^{-3}\,.$$

From radiative pion decay $\pi^+ \rightarrow e^+ + \nu_e + \gamma$:

$$-2.2 \times 10^{-3} < \frac{\alpha_{LL}}{a_{LL}^{SM}} < 2.72 \times 10^{-3}\,.$$

Form factors

$$H_i \to H_i^0 = g_i\, O_i\,,$$

where

$$O_S = 1\,, O_V = \gamma_\mu\,,\;\; O_A = \gamma_\mu\gamma_5\,,\;\; O_T = \sigma_{\mu\nu}\,,$$

$$g_V = F_1(q^2 \approx 0) = 1,\ g_A = G_A(q^2 \approx 0),\ g_{S,T} = g_{S,T}(q^2 \approx 0).$$

QCD lattice calculations:
$1.1 < g_A < 1.34$, $g_S = 0.8 \pm 0.4$ and $g_T = 1.05 \pm 0.35$.

Parameters

$$\lambda = g_A \frac{a_{LL} - a_{LR}}{a_{LL} + a_{LR}},$$

$$V_{RL} = \frac{a_{LL}a_{RL} - a_{LR}a_{RR}}{a_{LL}^2 - a_{LR}^2},$$

$$V_{RR} = \frac{a_{LL}a_{RR} - a_{LR}a_{RL}}{a_{LL}^2 - a_{LR}^2},$$

$$S_{kl} = g_S \frac{A_{kl}}{a_{LL} + a_{LR}},$$

$$T_{kk} = g_T \frac{\alpha_{kk}}{a_{LL} + a_{LR}},$$

$$s_L = S_{LL} + S_{LR}, \quad s_R = S_{RR} + S_{RL}.$$

Least Squares Analysis

$$\chi^2 = \sum_i \left[\frac{a_i - a}{\delta a_i}\right]^2 + \sum_j \left[\frac{A_j - A}{\delta A_j}\right]^2 + \sum_k \left[\frac{B_k - B}{\delta B_k}\right]^2 ,$$

$$\chi^2(\lambda, V_{RL}, V_{RR}, s_R, T_{RR}) =$$
$$\chi^2(\lambda, -V_{RL}, -V_{RR}, -s_R, -T_{RR}).$$

Data Selection

PAR.	VALUE	ERROR	PAPER ID
a	-0.1054	0.0055	BYRNE 02
	-0.1017	0.0051	STRATOWA 78
A	-0.11954	0.00112	MENDENHALL 12
	-0.11996	0.00058	MUND 12
	-0.11942	0.00166	LIU 10, MENDENHALL 12
	-0.1189	0.0007	ABELE 02
B	0.980	0.005	SCHUMANN 07
	0.967	0.012	KREUZ 05
	0.9801	0.0046	SEREBROV 98
	0.9894	0.0083	KUZNETSOV 95

All PAPER ID names are from PDG 2012, except the new ones:

MENDENHALL 12: M. P. Mendenhall et al., arXiv:1210.7048

MUND 12: D. Mund et al., arXiv:1204.0013

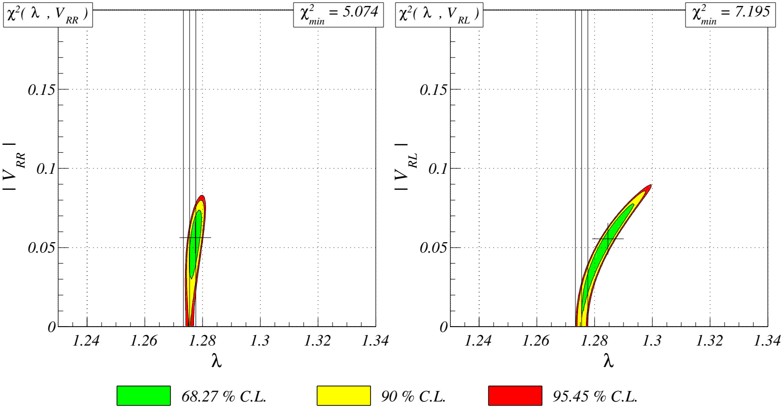
One-parameter Fit

$$a = -\frac{\lambda^2 - 1}{3\lambda^2 + 1},$$
$$A = -\frac{2\lambda(\lambda - 1)}{3\lambda^2 + 1},$$
$$B = \frac{2\lambda(\lambda + 1)}{3\lambda^2 + 1}.$$

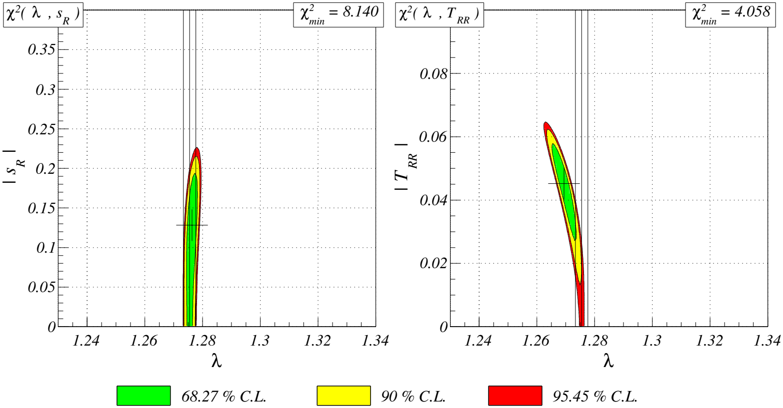
We obtained $\chi^2_{min} = 9.542$ (the value of χ^2 at minimum) with

$$\lambda = 1.2755 \pm \begin{cases} 0.0011 \text{ (68.27\% C.L.)}, \\ 0.0018 \text{ (90\% C.L.)}, \\ 0.0022 \text{ (95.45\% C.L.)}. \end{cases}$$

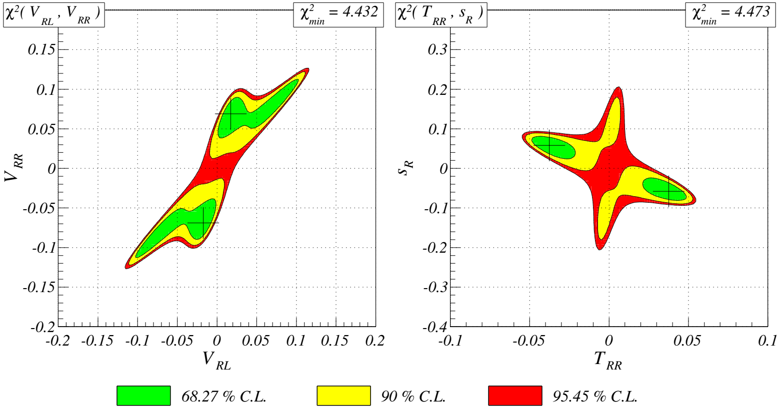
Two-parameter Fits



Two-parameter Fits



Two-parameter Fits ($\lambda_{SM} = \lambda = 1.2755$)



Statistical operator (CM frame)

See *e.g.* : R. Szafron, M. Zralek, Prog. Part. Nucl. Phys. 64 (2010) 210
M. Ochman, R. Szafron, M. Zralek, Nucl. Phys. Proc. Suppl. 217 (2011) 347
R. Szafron, M. Zralek, Phys. Lett. B 718 (2012) 113

$$\rho = \sum_{i,k=1,2,3} \sum_{\lambda,\delta=\pm 1} \int_0^{\Omega_\nu^{max}} d\Omega_\nu \int_0^{E_\nu^{max}} dE_\nu \frac{d\rho_{\lambda,i;\delta,k}}{dE_\nu d\Omega_\nu} |\bar{\nu}_i(\lambda, \mathbf{p}_\nu)\rangle\langle\bar{\nu}_k(\delta, \mathbf{p}_\nu)| \, ,$$

$$d\rho_{\lambda,i;\lambda,k} = \frac{1}{N} \sum_{\lambda_X,\lambda_Y,\lambda_e} \frac{d^3\mathbf{p}_Y}{(2\pi)^3 2E_Y} \frac{d^3\mathbf{p}_e}{(2\pi)^3 2E_e} \frac{d^3\mathbf{p}_\nu}{(2\pi)^3 2E_\nu} \times (2\pi)^4 \delta^{(4)}(p_X - p_Y - p_e - p_\nu) A_{\lambda,i} A^*_{\lambda,k} \, ,$$

$$\sum_{i=1}^{3} \sum_{\lambda=\pm 1} \int_0^{\Omega_\nu^{max}} d\Omega_\nu \int_0^{E_\nu^{max}} dE_\nu \frac{d\rho_{\lambda,i;\lambda,i}}{dE_\nu d\Omega_\nu} = 1 \, .$$

Allowed β^- transitions

Allowed transitions = no orbital angular momentum change: M_{GT} – Gamow-Teller $(S_Y - S_X = 0, \pm 1$ except $S_Y = S_X = 0)$, M_F – Fermi $(S_Y = S_X)$

$$\frac{d\rho_{+,i;+,k}}{dE_\nu d\Omega_\nu} = U^L_{ei}\left(U^L_{ek}\right)^* \rho_{+,+}(E_\nu)\,, \quad \frac{d\rho_{-,i;-,k}}{dE_\nu d\Omega_\nu} = U^R_{ei}\left(U^R_{ek}\right)^* \rho_{-,-}(E_\nu)$$

$$\rho_{+,+}(E_\nu) = E_\nu^2 E_e |\mathbf{p}_e| \frac{|M_{GT}|^2 u_L + |M_F|^2 v_L}{|M_{GT}|^2(\bar{u}_L + \bar{u}_R) + |M_F|^2(\bar{v}_L + \bar{v}_R)}\,,$$

$$\rho_{-,-}(E_\nu) = E_\nu^2 E_e |\mathbf{p}_e| \frac{|M_{GT}|^2 u_R + |M_F|^2 v_R}{|M_{GT}|^2(\bar{u}_L + \bar{u}_R) + |M_F|^2(\bar{v}_L + \bar{v}_R)}\,,$$

Reduced nuclear matrix elements (Wigner-Eckart theorem):
– for pure Fermi transitions: $|M_F|^2 \neq 0$, $|M_{GT}|^2 = 0$,
– for pure Gamow-Teller transitions: $|M_F|^2 = 0$, $|M_{GT}|^2 \neq 0$.

Allowed β^- transitions

$$u_L = 4T_{LL}^2 - 4\lambda T_{LL}\frac{m_e}{E_e} + \lambda^2 \,,$$

$$u_R = \lambda^2 \left(V_{RR} - V_{RL}\right)^2 - 4\lambda T_{RR} \left(V_{RR} - V_{RL}\right) \frac{m_e}{E_e} + 4T_{RR}^2 \,,$$

$$v_L = s_L^2 + 2s_L\frac{m_e}{E_e} + 1 \,,$$

$$v_R = \left(V_{RR} + V_{RL}\right)^2 + 2s_R \left(V_{RR} + V_{RL}\right) \frac{m_e}{E_e} + s_R^2 \,.$$

$$\bar{w} = \int_0^{\Omega_\nu^{max}} d\Omega_\nu \int_0^{E_\nu^{max}} dE_\nu E_\nu^2 E_e |\mathbf{p}_e| w$$

Statistical operator (LAB frame)

$$\rho' = \sum_{i,k=1,2,3} \sum_{\lambda,\delta=\pm 1} \int_0^{\Omega_\nu'^{max}} d\Omega_\nu' \int_0^{E_\nu'^{max}(\Omega_\nu')} dE_\nu' \frac{d\rho_{\lambda,i;\delta,k}}{dE_\nu' d\Omega_\nu'} |\bar{\nu}_i(\lambda, \mathbf{p}_\nu')\rangle\langle\bar{\nu}_k(\delta, \mathbf{p}_\nu')| \, ,$$

$$\mathbf{p}_\nu \approx \mathbf{p}_{\nu,z} = (0, 0, E_\nu),$$

$$\rho' \approx \bar{\rho} = \sum_{i,k=1,2,3} \sum_{\lambda=\pm 1} \int_0^{E_\nu'^{max}} dE_\nu' \frac{d\bar{\rho}_{\lambda,i;\lambda,k}}{dE_\nu'} |\bar{\nu}_i(\lambda, \mathbf{p}_{\nu,z}')\rangle\langle\bar{\nu}_k(\lambda, \mathbf{p}_{\nu,z}')| \, ,$$

$$\frac{d\bar{\rho}_{\lambda,i;\lambda,k}}{dE_\nu'} = 2\pi \int_{\cos\theta_\nu'^{max}}^{1} d\cos\theta_\nu' \, \frac{d\rho_{\lambda,i;\lambda,k}}{d\cos\theta_\nu' \, dE_\nu'} \, ,$$

$$\frac{d\rho_{\lambda,i;\lambda,k}}{d\cos\theta_\nu' \, dE_\nu'} = \frac{d\rho_{\lambda,i;\lambda,k}}{d\cos\theta_\nu \, dE_\nu} |\det J| \, .$$

Statistical operator (LAB frame)

$$\frac{d\bar{\rho}_{+,i;+,k}}{dE'_\nu} = U^L_{ei}\left(U^L_{ek}\right)^* 2\pi\left(1-\cos\theta^{max}_\nu\right)$$
$$\times(\gamma-\sqrt{\gamma^2-1})\rho_{+,+}\left((\gamma-\sqrt{\gamma^2-1})E'_\nu\right) ,$$
$$\frac{d\bar{\rho}_{-,i;-,k}}{dE'_\nu} = U^R_{ei}\left(U^R_{ek}\right)^* 2\pi\left(1-\cos\theta^{max}_\nu\right)$$
$$\times(\gamma-\sqrt{\gamma^2-1})\rho_{-,-}\left((\gamma-\sqrt{\gamma^2-1})E'_\nu\right) ,$$

$$\cos\theta^{max}_\nu = \frac{\sqrt{\gamma^2-1}-\gamma\cos\theta'^{max}_\nu}{\sqrt{\gamma^2-1}\cos\theta'^{max}_\nu-\gamma} .$$

$$\sum_{i=1}^{3}\sum_{\lambda=\pm 1}\int_0^{E'^{max}_\nu} dE'_\nu \frac{d\bar{\rho}_{\lambda,i;\lambda,i}}{dE'_\nu} = 1 .$$

Flux

The density of the flux:

$$j_\lambda(E'_\nu) = \frac{N_P}{\pi D^2} \frac{1 - \cos\theta_\nu^{max}}{2} \sum_{i=1}^{3} \frac{d\bar{\rho}_{\lambda,i;\lambda,i}}{dE'_\nu} \,,$$

The total density of the flux:

$$j(E'_\nu) = \sum_{\lambda=\pm 1} j_\lambda(E'_\nu) \,.$$

Evolution of the state

$$\rho'(\mathbf{L}, T) = \mathcal{U}(\mathbf{L}, T)\rho' \mathcal{U}^{+}(\mathbf{L}, T)\,.$$

$$\mathcal{U}(\mathbf{L}, T) = \exp(-i\mathcal{P}^{\mu} X_{\mu})$$

with $X = (T, \mathbf{L})$

$$\mathcal{P}^{\mu}|\bar{\nu}_i(\lambda, \mathbf{p}_{\nu})\rangle = p_i^{\mu}|\bar{\nu}_i(\lambda, \mathbf{p}_{\nu})\rangle\,,$$

where $p_i = (E_i, \mathbf{p}_{\nu})$, $E_i = \sqrt{\mathbf{p}_{\nu}^2 + m_i^2}$.

Evolution of the state

Let us now consider $\mathbf{L} = (0, 0, L)$ and $\mathbf{p}_\nu = \mathbf{p}_{\nu,z} = (0, 0, E_\nu)$. We take $T \approx L$ and $E_i \approx E_\nu + m_i^2/(2E_\nu)$ so

$$\bar{\rho}(L) = \sum_{i,k=1,2,3} \sum_{\lambda=\pm 1} \int_0^{E_\nu^{max}} dE_\nu \frac{d\bar{\rho}_{\lambda,i;\lambda,k}(L)}{dE_\nu} |\bar{\nu}_i(\lambda, \mathbf{p}_{\nu,z})\rangle\langle\bar{\nu}_k(\lambda, \mathbf{p}_{\nu,z})| \,,$$

where

$$\frac{d\bar{\rho}_{\lambda,i;\lambda,k}(L)}{dE_\nu} = \frac{d\bar{\rho}_{\lambda,i;\lambda,k}}{dE_\nu} \exp\left(-i\frac{\Delta m_{ik}^2}{2E_\nu}L\right)$$

with $\Delta m_{ik}^2 = m_i^2 - m_k^2$.

Detection

$$N_\ell = \int_{E_\nu^{th,\ell}}^{E_\nu^{max}} N_\ell(E_\nu) dE_\nu \, ,$$

$$N_\ell(E_\nu) = n_D t_D \frac{N_P}{\pi D^2} \frac{1 - \cos\theta_\nu^{max}}{2} \sigma_{e\to\ell}(E_\nu, L)$$

$$\sigma_{e\to\ell}(E_\nu, L) = \sum_{\lambda,\lambda_p,\lambda_n,\lambda_\ell=\pm1} \sum_{i,k=1,2,3} \frac{1}{4\sqrt{(p_\nu p_p)^2}} \frac{1}{2}$$

$$\times \int \frac{d^3\mathbf{p}_\ell}{(2\pi)^3 2E_\ell} \int \frac{d^3\mathbf{p}_n}{(2\pi)^3 2E_n}$$

$$\times (2\pi)^4 \delta^{(4)}(p_p + p_\nu - p_n - p_\ell) A^{D,\ell}_{\lambda,i} \frac{d\bar{\rho}_{\lambda,i;\lambda,k}(L)}{dE_\nu} \left(A^{D,\ell}_{\lambda,k}\right)^* ,$$

Detection

$$A^{D,\ell}_{\lambda=+1,i} = \left(U^L_{\ell i}\right)^* M^{D,\ell}_{\lambda=+1}\,, \quad A^{D,\ell}_{\lambda=-1,i} = \left(U^R_{\ell i}\right)^* M^{D,\ell}_{\lambda=-1}\,,$$

$$N_\ell(E_\nu) = N_{+;\ell}(E_\nu) + N_{-;\ell}(E_\nu)\,,$$

where

$$N_{+;\ell}(E_\nu) = n_D t_D j_+(E_\nu)\sigma_{+;\ell}(E_\nu) P^L_{e\to\ell}(E_\nu, L)\,,$$
$$N_{-;\ell}(E_\nu) = n_D t_D j_-(E_\nu)\sigma_{-;\ell}(E_\nu) P^R_{e\to\ell}(E_\nu, L)$$

$$P^j_{e\to\ell}(E_\nu, L) = \sum_{i,k=1,2,3} U^j_{ei} U^j_{\ell k} \left(U^j_{ek}\right)^* \left(U^j_{\ell i}\right)^* \exp\left(-i\frac{\Delta m^2_{ik}}{2E_\nu} L\right)$$

Cross–sections

$$\sigma_{\pm;\ell}(E_\nu) = \int_{Q^2_{min}(E_\nu)}^{Q^2_{max}(E_\nu)} dQ^2 \frac{d\sigma_{\pm;\ell}(E_\nu)}{dQ^2} \,,$$

The target protons in the detector are at rest.

$$Q^2_{min}(E_\nu) = \frac{2E_\nu^2 m_N - m_N m_\ell^2 - z}{2E_\nu + m_N} \,,$$

$$Q^2_{max}(E_\nu) = \frac{2E_\nu^2 m_N - m_N m_\ell^2 + z}{2E_\nu + m_N}$$

with

$$z = E_\nu m_\ell^2 - E_\nu \sqrt{(s - m_\ell^2)^2 - 2(s + m_\ell^2) m_N^2 + m_N^4}$$

and $s = m_N^2 + 2E_\nu m_N$

Cross–sections

C. H. Llewellyn Smith, Phys. Rept. 3 (1972) 261

$$\frac{d\sigma_\ell^{SM}}{dQ^2} = \frac{(G_\beta^{SM})^2 m_N^2}{4\pi E_\nu^2}\left[A - \frac{s-u}{m_N^2}B + \left(\frac{s-u}{m_N^2}\right)^2 C\right],$$

where $G_\beta^{SM} = V_{ud}\, G_F/\sqrt{2}$, $s-u = 4m_N E_\nu - Q^2 - m_\ell^2$ and

$$A = \frac{m_\ell^2 + Q^2}{m_N^2}\Big\{(1+r)G_A^2 - (1-r)F_1^2 + r(1-r)F_2^2 + 4rF_1F_2$$
$$- \frac{m_\ell^2}{4m_N^2}\left[(F_1+F_2)^2 + (G_A+2G_P)^2 - 4(r+1)G_P^2\right]\Big\},$$
$$B = 4rG_A(F_1+F_2)\,,$$
$$C = \frac{1}{4}(G_A^2 + F_1^2 + rF_2^2)$$

with $r = Q^2/(4m_N^2)$, $F_{1,2} \equiv F_{1,2}(Q^2)$, $G_{A,P} \equiv G_{A,P}(Q^2)$.

Cross–sections

$$\frac{d\sigma_{+;\ell}}{dQ^2} = \frac{d\sigma_\ell^{SM}}{dQ^2}(G_\beta^{SM} \to G_\beta, G_{A,P} \to v_a G_{A,P})\,,$$
$$\frac{d\sigma_{-;\ell}}{dQ^2} = \frac{d\sigma_\ell^{SM}}{dQ^2}(G_\beta^{SM} \to G_\beta, F_{1,2} \to v_+ F_{1,2}, G_{A,P} \to v_a v_- G_{A,P})\,.$$

$$v_a = \frac{a_{LL} - a_{LR}}{a_{LL} + a_{LR}}\,, \quad v_\pm = \frac{a_{RR} \pm a_{RL}}{a_{LL} \pm a_{LR}}\,.$$

Numerical results

$$\Delta N_{e\mu}(E_\nu) = 1 - \frac{N_\mu^{NP}(E_\nu)}{N_e^{NP}(E_\nu)} \Bigg/ \frac{N_\mu^{SM}(E_\nu)}{N_e^{SM}(E_\nu)} \,,$$

$$\Delta N_{e\mu}(E_\nu) \equiv \Delta N_{e\mu}(E_\nu; \Theta^L, \Theta^R) \,,$$

where $\Theta^k \equiv (\theta_{12}^k, \theta_{13}^k, \theta_{23}^k, \delta_{CP}^k)$.

$$U^k = \begin{pmatrix} e^{i\alpha_1^k} & 0 & 0 \\ 0 & e^{i\alpha_2^k} & 0 \\ 0 & 0 & e^{i\alpha_3^k} \end{pmatrix} U(\theta_{12}^k, \theta_{13}^k, \theta_{23}^k, \delta_{CP}^k) \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha_4^k} & 0 \\ 0 & 0 & e^{i\alpha_5^k} \end{pmatrix},$$

where $U \equiv U(\theta_{12}, \theta_{13}, \theta_{23}, \delta_{CP})$ as in the PDG 2012:

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{CP}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{CP}} & c_{23}c_{13} \end{pmatrix}$$

$c_{ij} \equiv \cos\theta_{ij}$, $s_{ij} \equiv \sin\theta_{ij}$

Numerical results

We calculate $\Delta N_{e\mu}(E_\nu)$ varying the $\theta^L_{ij} \equiv \theta_{ij} \in [0, \pi/2]$ in the limits recommended by the PDG 2012:

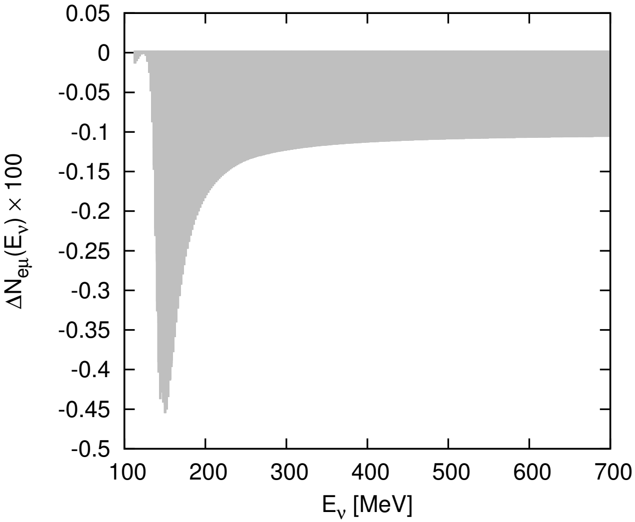
$$\begin{aligned} \sin^2(2\theta_{12}) &= 0.857 \pm 0.024\,, \\ \sin^2(2\theta_{23}) &> 0.95\,, \\ \sin^2(2\theta_{13}) &= 0.098 \pm 0.013 \end{aligned}$$

and with $\delta^L_{CP} = 0$, while $\theta^R_{ij} \in [0, \pi/2]$ and $\delta^R_{CP} \in [0, 2\pi]$. The mass differences we set to (PDG 2012):

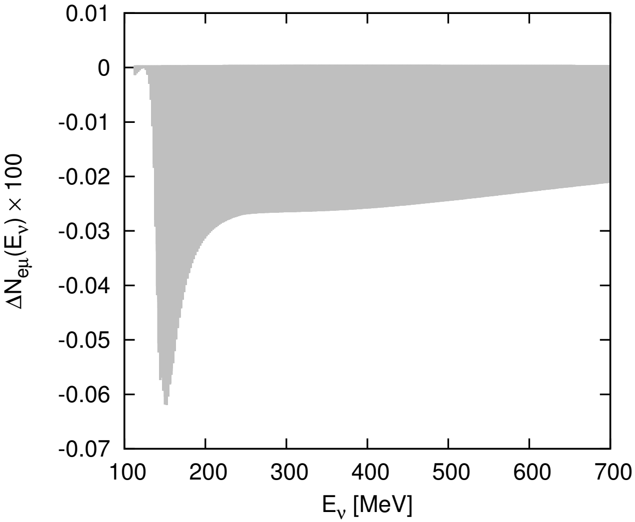
$$\begin{aligned} \Delta m^2_{21} &= (7.50 \pm 0.20) \times 10^{-5}\,\mathrm{eV}^2\,, \\ |\Delta m^2_{32}| &= 0.00232^{+0.00012}_{-0.00008}\,\mathrm{eV}^2\,, \end{aligned}$$

we choose $\Delta m^2_{32} > 0$.

Numerical results: $a_{RR}/a_{LL} = 0.06$, ${}^6\mathrm{He}$ accelerated to $\gamma = 100$, $L = 130\,\mathrm{km}$



Numerical results: $a_{RL}/a_{LL} = 0.03$, $^6\mathrm{He}$ accelerated to $\gamma = 100$, $L = 130\,\mathrm{km}$



Conclusions

1. Effects of New Physiscs are to small to observe (systematic errors of the signal and the backgrounds were estimated to be not smaller than 2% — see *e.g.* M. Mezzetto, J. Phys. G 29 (2003) 1771, M. Mezzetto, Nucl. Phys. Proc. Suppl. 155 (2006) 214 vs. calculated effects of NP — below 0.5%).
2. Standard Model formulas can be safely used.