Higgs boson production at N^3LO

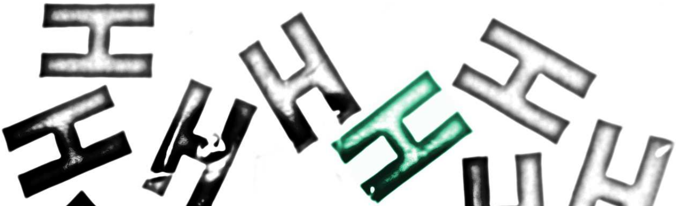
– current status and developments

Jens Hoff (DESY)

12th of March, 2015

Based on work in collaboration with:

- M. Höschele, A. Pak, M. Steinhauser, T. Ueda
- + C. Anzai, A. Hasselhuhn, W. Kilgore (in preparation)

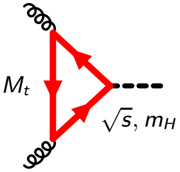


Introduction

- Higgs boson:
  $m_H = 125$ GeV, spin 0
- Couplings to bosons and fermions:
  experimental precision $\mathcal{O}(10\% - 20\%)$

Needed: precise theory predictions

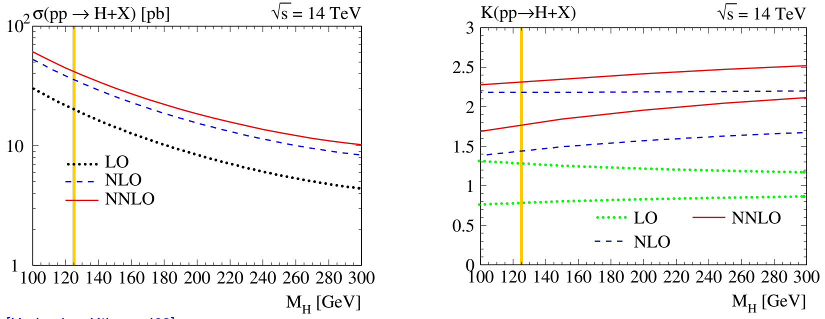
- LHC: gluon fusion dominant production process
  $\Longrightarrow$ QCD corrections!
- Our interest: total inclusive production cross section



- Soft: $x = m_H^2/s \to 1$

Motivation

Large corrections



[Harlander, Kilgore; '02]

- Large corrections:

$$\sigma_{\text{NNLO}} = \sigma_{\text{LO}} + \overbrace{\delta\sigma_{\text{NLO}}}^{\sim 80\%} + \overbrace{\delta\sigma_{\text{NNLO}}}^{\sim 20\%} \quad \Longrightarrow \quad \sim 100\%$$

- Large uncertainty:

$$K_{\text{NLO}} \approx 1.8 \pm 0.4, \; K_{\text{NNLO}} \approx 2.0 \pm 0.3 \quad \Longrightarrow \quad \sim 15\%$$

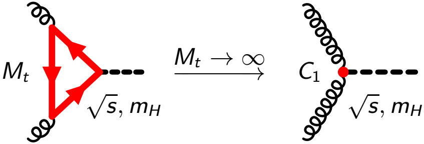
- Due to PDFs + higher orders (partonic)

Methods

Effective field theory (EFT)

- Loop-induced process, dominated by top quark mass
- Effective field theory with top quark integrated out:

$$\mathcal{L}_{Y,\text{eff}} = -\frac{H}{v} C_1 \mathcal{O}_1 \quad \text{and} \quad \mathcal{O}_1 = \frac{1}{4} G^a_{\mu\nu} G^{a,\mu\nu}$$



- Reduced numbers of scales and loops
- Finite matching coefficient C_1, needed to 4-loop
  [Chetyrkin, Kniehl, Steinhauser; '98] [Schröder, Steinhauser; '06] [Chetyrkin, Kühn, Sturm; '06]
- Corrections due to m_H^2/M_t^2 at NNLO of $\mathcal{O}(1\%)$
  [Pak, Rogal, Steinhauser; '09-'11] [Harlander, Mantler, Marzani, Ozeren; '09-'10]

  $\Longrightarrow$ $\boxed{\text{N}^3\text{LO within EFT}}$
- UV renormalization of α_s and $\mathcal{O}_1$ to 3-loop
  [Tarasov, Vladimirov, Zharkov; '80] [Larin, Vermaseren; '93] [Spiridonov]

Methods

Higher-order corrections:

Virtual More loops

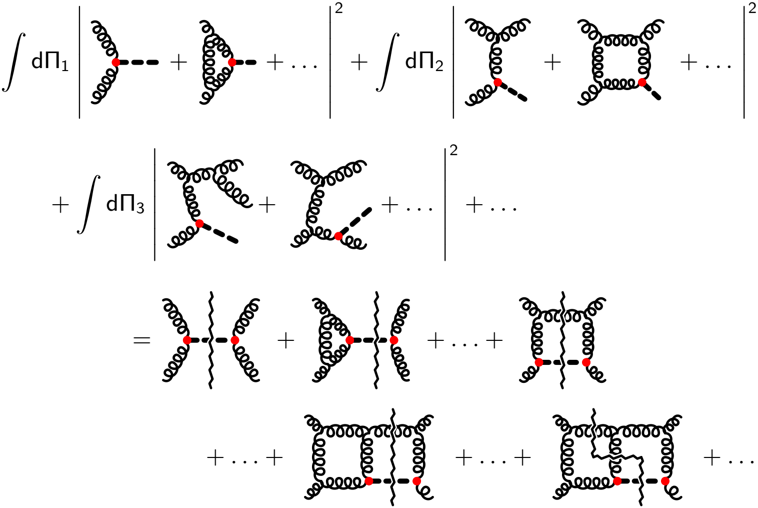
Real Additional final state particles (different phase spaces)

$$\sigma(i \to f) \quad \sim \quad \sum_f \int \mathrm{d}\Pi_f \; |\mathcal{M}(i \to f)|^2 \quad \sim \quad \mathrm{Im}\, \mathcal{M}(i \to i)$$

Methods

Optical theorem and Cutkosky's rules

E.g. to NNLO:



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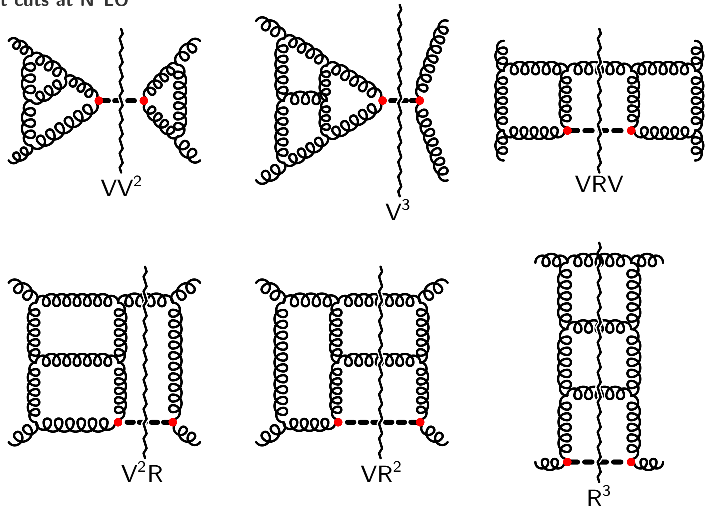
Pros
- Forward scattering $\Rightarrow$ simplified kinematics
- Common treatment of loop and phase space integrals
- Calculation of Im only for master integrals

Cons
- More loops, more diagrams
- Only total cross section

Approach first used in [Anastasiou, Melnikov; '02]

Anatomy of the total cross section

Different cuts at N^{3}LO

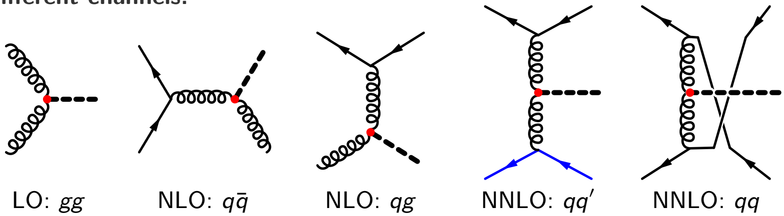


Singular behavior with k partons:

$$\left\{(1-x)^{-2\epsilon}, (1-x)^{-3\epsilon}, \ldots, (1-x)^{-2k\epsilon}\right\}$$

Anatomy of the total cross section

Different channels:



Correspondence: channels and cuts (and singular behavior)

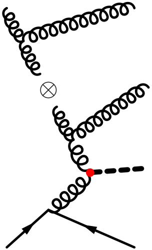
- qq', qq $\longrightarrow$ VR^2, R^3
- $q\bar{q}$, qg $\longrightarrow$ + VRV, V^{2}R
- gg $\longrightarrow$ + VV^2, V^3

General form of result:

$$\sigma_{ij}^{(k)} = \delta(1-x)\ \sigma_{ij}^{(k,-1),\mathrm{S}} + \sum_{l=0}^{2k-1} \left[\frac{\ln^l(1-x)}{1-x}\right]_+ \sigma_{ij}^{(k,l),\mathrm{S}} + \sum_{l=0}^{2k-1} \ln^l(1-x)\ \sigma_{ij}^{(k,l),\mathrm{R}}$$

Existing results

- LO calculation (exact)
  [Ellis et al.; '76] [Wilczek et at.; '77] [Georgi et al.; '78] [Rizzo; '80]
- NLO (exact)
  [Dawson; '91] [Djouadi, Spira, Zerwas; '91]
- NNLO (EFT) $\Longrightarrow$ Soft expansion to 3rd order valid $\mathcal{O}(1\%)$
  [Harlander, Kilgore; '02] [Anastasiou, Melnikov; '02] [Ravindran, Smith, van Neerven; '03]
- NNLO (m_H^2/M_t^2 corrections) $\Longrightarrow$ $\mathcal{O}(1\%)$
  [Pak, Rogal, Steinhauser; '09-'11] [Harlander, Mantler, Marzani, Ozeren; '09-'10]
- N^3LO IR counterterms
  - 3-loop splitting functions
    [Moch, Vermaseren, Vogt; '02]
  - NNLO master integrals to higher orders in ϵ
    [Pak, Rogal, Steinhauser; '11] [Anastasiou, Bühler, Duhr, Herzog; '12]
  - Cross sections and convolution integrals
    [Höschele, JH, Pak, Steinhauser, Ueda; '12, '13] [Bühler, Lazopoulos; '13]
- N^3LO scale variation $\Longrightarrow$ $\mathcal{O}(2\% - 8\%)$
  [Bühler, Lazopoulos; '13]



Existing results

- N^3LO corrections
  - VV^2 and V^3 – 3-loop gluon form factor
    [Baikov, Chetyrkin, Smirnov^2, Steinhauser; '09] [Gehrmann, Glover, Huber, Ikizlerli, Studerus; '09]
  - VRV exact in x
    [Anastasiou, Duhr, Dulat, Herzog, Mistlberger; '13] [Kilgore; '14]
  - V^2R exact in x
    [Dulat, Mistlberger; '14] [Duhr, Gehrmann, Jaquier; '14]
  - VR^2 expansion in $x \to 1$
    [Anastasiou, Duhr, Dulat, Furlan, Gehrmann; '14] [Li, von Manteuffel, Schabinger, Zhu; '14]
  - R^3 expansion in $x \to 1$
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  - Particular contribution to R^3 exact in x
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  - 1st and 2nd term in the $x \to 1$ expansion
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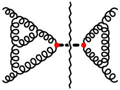
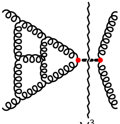
    - $\sigma_{ij}^{(3,-1),S}$, $\sigma_{ij}^{(3,l),S}$, $\sigma_{ij}^{(3,0),R}$
    - $\sigma_{ij}^{(3,l),R}$, $l \in \{3,4,5\}$

    $\Longrightarrow$ Soft expansion not reliable!?





- Many different resummations
  [Catani et al.] [Moch et al.] [Ahrens et al.] [de Florian et al.] [Ball et al.] [...]

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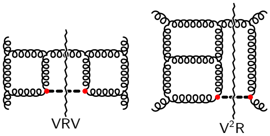

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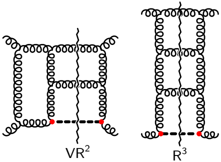

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Complexity of the calculation

Numbers of diagrams from LO to NNLO

Order	Channel	Number of diagrams (fermionic loops)		
LO	gg	$\#1$		
NLO	$q\bar{q}$	$\#1$		
	qg	$\#1$		
	gg_{virt}	$\#10$		
	gg_{real}	$\#38$		
	$\sum$	$\#50$		
NNLO	qq'	$\#1$		
	qq	$\#2$		
	$q\bar{q}$	$\#84$	$= \#81$	$+\#3\ n_l$
	qg	$\#124$	$= \#122$	$+\#2\ n_l$
	gg_{virt}	$\#294$	$= \#252$	$+\#42\ n_l$
	gg_{real}	$\#2\,458$	$= \#2\,293$	$+\#165\ n_l$
	$\sum$	$\#2\,964$	$= \#2\,752$	$+\#212\ n_l$

Complexity of the calculation

Number of diagrams at N^3LO

Channel	Number of diagrams (fermionic loops)			
qq'	$\#220$	$= \#216$	$+\#4\ n_l$	
qq	$\#404$	$= \#396$	$+\#8\ n_l$	
$q\bar{q}$	$\#4\,889$	$= \#4\,438$	$+\#445\ n_l$	$+\#6\ n_l^2$
qg	$\#9\,591$	$= \#8\,976$	$+\#612\ n_l$	$+\#3\ n_l^2$
gg_{virt}	$\#9\,538$	$= \#7\,266$	$+\#2\,180\ n_l$	$+\#92\ n_l^2$
gg_{real}	$\#150\,246$	$= \#128\,676$	$+\#21\,196\ n_l$	$+\#374\ n_l^2$
$\sum$	$\#174\,938$	$= \#150\,014$	$+\#24\,449\ n_l$	$+\#475\ n_l^2$

Complexity of the calculation

Numbers of diagrams, topologies and master integrals

Number of	LO	NLO	NNLO	N^3LO
Diagrams	1	50	2 946	174 938
Topologies	0	1	11	188
Master integrals	0	1	20	$\mathcal{O}(300)$

- # diagrams: not really crucial (computing time ...)
- # topologies: automatization $\Longrightarrow$ `TopoID`
- # master integrals: real challenge

General procedure

1. Reduction of integrals with full x-dependence
   $\longrightarrow$ only contributing cuts
2. Construct differential equations for master integrals
   $\longrightarrow$ canonical basis (in general: coupled system)
3. Soft limit $x \to 1$ as boundary condition
   $\longrightarrow$ leading term using Mellin-Barnes representation

Canonical differential equations:

$$\frac{\mathrm{d}}{\mathrm{d}x} f_i(x, \epsilon) = \epsilon \, A_{ij}(x) \; f_j(x, \epsilon)$$

[Henn et al.; '13-. . .]

- ϵ- and x-dependence factorize
- Solve order-by-order in ϵ
- A_{ij}: alphabet of appearing functions

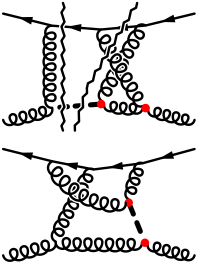
E.g. HPLs:

$$\mathrm{H}_{\vec{w}}(x) = \int_0^x \mathrm{d}x' \, f_{w_1}\left(x'\right) \mathrm{H}_{\vec{w}_{n-1}}\left(x'\right) \quad \text{and} \quad f_0(x) = \frac{1}{x}, \; f_1(x) = \frac{1}{1-x}, \; f_{-1}(x) = \frac{1}{1+x}$$

Toolchain

Reduction (step 1)

1. Generate Feynman diagrams
   QGRAF [Nogueira; '93]
2. Select diagrams with specific cuts
   filter [JH, Pak; (unpublished)]
3. Map diagrams to topologies (← graph information)
   exp [Harlander, Seidensticker, Steinhauser; '98]
   reg [Pak; (unpublished)]
4. Reduction to scalar integrals (← generic topologies)
   FORM [Kuipers, Ueda, Vermaseren, Vollinga; '13]
5. Reduction to master integrals (← basic topologies)
   rows [JH, Pak; (unpublished)]
   FIRE [Smirnov]
6. Minimal basis of master integrals
   TopoID [JH, Pak; (unpublished)]



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Idea: Generic, process independent `Mathematica` package.

Feynman diagrams $\longrightarrow$ reduced result

Features

- **Topo**logy **Id**entification, minimal sets
- Distinct and scaleless subtopologies
- Properties: completeness, linear dependence, etc.
- Partial fractioning relations
- Complete symmetrization
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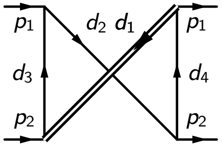
Generic topology at NLO:



$$= \int \mathrm{d}k_1{}^D \, \frac{1}{d_1^{a_1} d_2^{a_2} d_3^{a_3} d_4^{a_4}} = T(a_1, a_2, a_3, a_4)$$

$$\begin{aligned}
d_1 &= m_H^2 + k_1^2 \\
d_2 &= (p_1 + p_2 + k_1)^2 = -s + 2p_1 \cdot k_1 + 2p_2 \cdot k_1 + k_1^2 \\
d_3 &= (p_2 + k_1)^2 = 2p_2 \cdot k_1 + k_1^2 \\
d_4 &= (p_1 + k_1)^2 = 2p_1 \cdot k_1 + k_1^2
\end{aligned}$$

$\{d_i\}$**:**

- Linearly dependent or independent?
- Complete or incomplete?

(concerning scalar products $\{p_i \cdot k_j, k_i \cdot k_j\}$)

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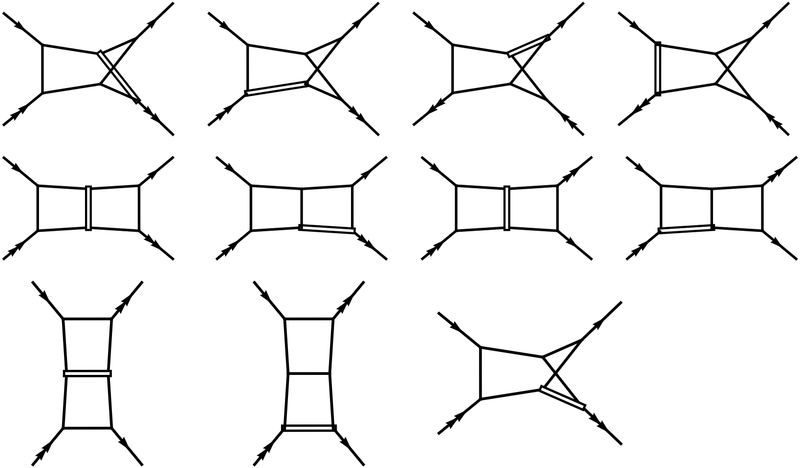

(sufficient for all 2 946 diagrams)

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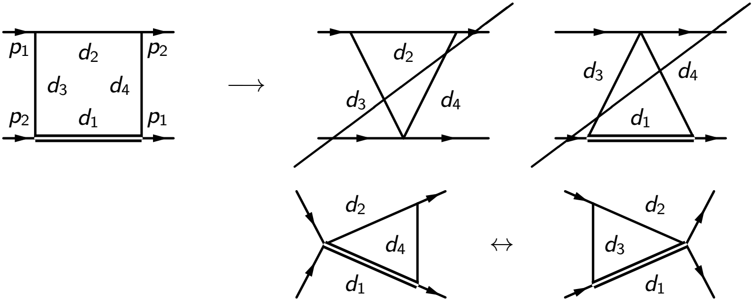
Partial fractioning at NLO:



via Gröbner basis:

$$
\begin{aligned}
d_4 &\to -m_H^2 + s + d_1 + d_2 - d_3 \\
\frac{d_3}{d_4} &\to \frac{1}{d_4}\left(-m_H^2 + s + d_1 + d_2 - d_4\right) \\
\frac{d_2}{d_3\,d_4} &\to \frac{1}{d_3\,d_4}\left(m_H^2 - s - d_1 + d_3 + d_4\right) \\
\frac{d_1}{d_2\,d_3\,d_4} &\to \frac{1}{d_2\,d_3\,d_4}\left(m_H^2 - s - d_2 + d_3 + d_4\right) \\
\frac{1}{d_1\,d_2\,d_3\,d_4} &\to \frac{1}{\left(m_H^2 - s\right)d_1\,d_2\,d_3\,d_4}\left(d_1 + d_2 - d_3 - d_4\right)
\end{aligned}
$$

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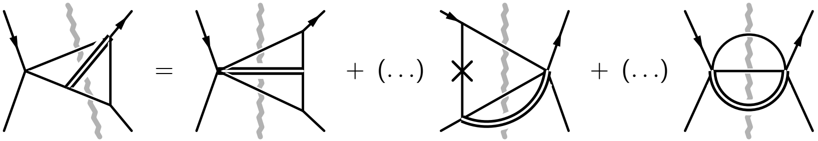
Non-trivial relation at NNLO:



- Cross-topology relations; not from Laporta reduction
- Simplify calculation
- Usefull cross-checks

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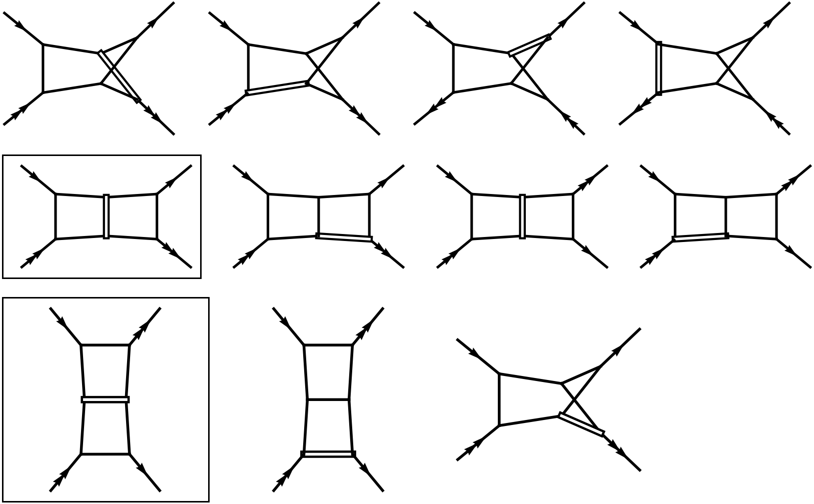

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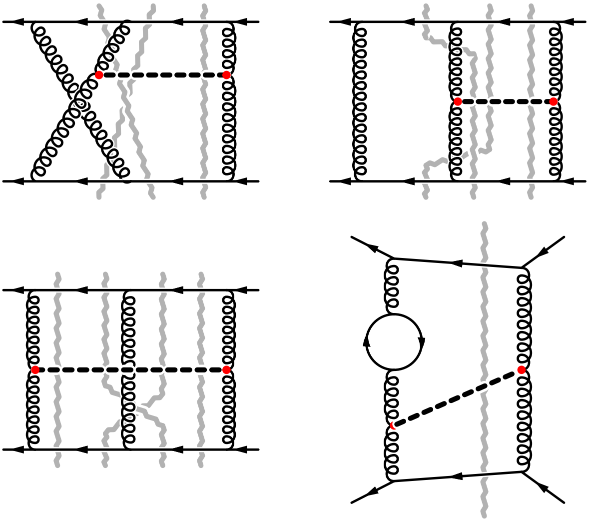
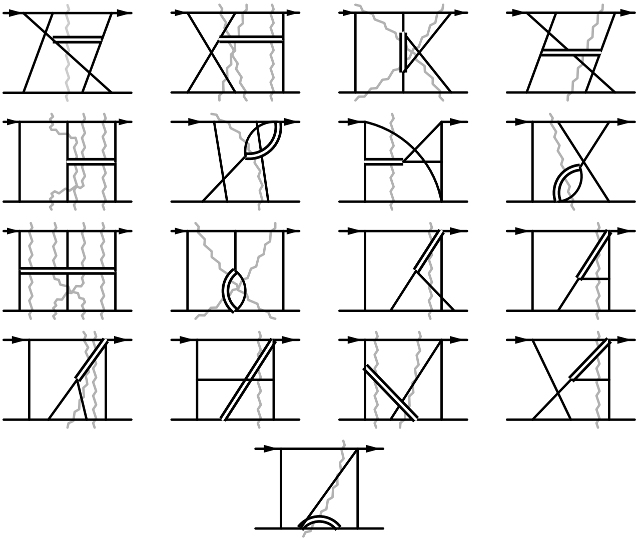
qq'-channel

220 **diagrams, e.g.**



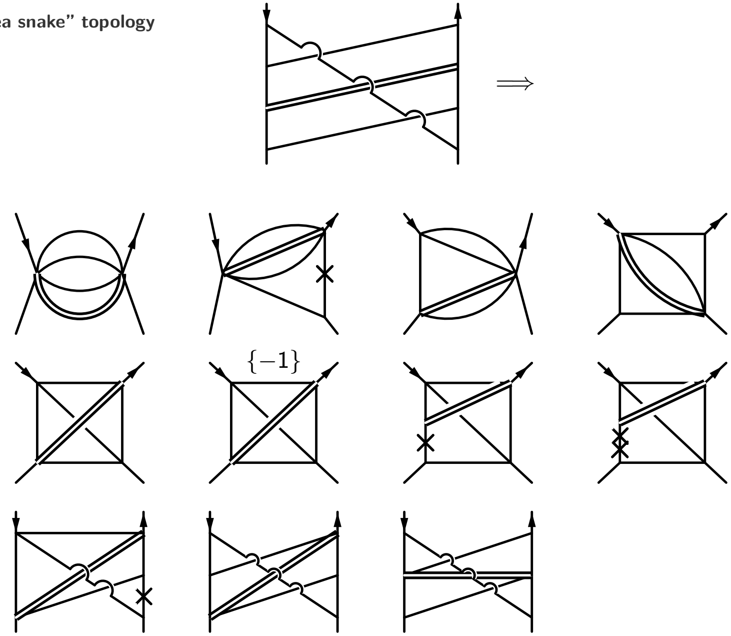
qq'-channel

17 topologies with 3- and 4-particle cuts



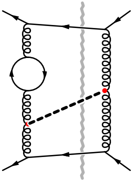
qq'-channel

E.g. “sea snake” topology



qq'-channel

- 220 diagrams
  $\Longrightarrow$ $\mathcal{O}(50\,000)$ integrals (including ξ-dependence)
- 17 topologies
  $\Longrightarrow$ $\mathcal{O}(2\,000\,000)$ relations (from IBPs)
- 332 master integrals from complete reduction ✓
- 108 master integrals (using non-trivial relations)
- Checks:
  - Cancellation of the gauge parameter
  - Finite fermionic contribution, i.e. diagrams of type



- Integrals with 3- and 4-particle cuts [Höschele, JH, Ueda; '14]
- Construction of canonical bases finished ✓
- Computation of boundaries almost completed (✓)
- HPLs not sufficient, alphabet includes $\boxed{1+4x}$ and $\boxed{x\sqrt{1+4x}}$

Conclusion

- For Higgs production at N^3LO needed:
  - exact dependence on $x = m_H^2/s$
  - more terms in soft expansion $x \to 1$
- Last missing pieces: $\boxed{VR^2}$ and $\boxed{R^3}$
- qq'-channel almost completed

- `TopoID`: process independent tool, many other applications
  - Higgs boson pair production
  - Drell-Yan process
  - Higgs boson + jet production
  - ...